I. THEORY

Sir Isaac Newton discovered bright and dark concentric rings when he held two lenses in contact. This phenomenon can only be explained as constructive and destructive interference, which requires a wave theory of light. Surprisingly, Newton failed to make the connection, and clung to his particle theory of light.

The purpose of this experiment is to test the consistency of the theoretical analysis of this phenomenon, and to measure the radius of curvature of a convex surface.

The apparatus consists of a plane glass plate held in contact with the convex surface of a plano-convex lens by a frame. Monochromatic light of wavelength $546.1 \text{ nm}$, from a mercury lamp with a green filter, reflects downward from a clear glass plate onto the apparatus. Some of the light reflects back upwards, passes through the plate and through a traveling microscope, and enters the eye of the experimenter. Note that the curvature of the lens in the figures is exaggerated in order to show detail.

Consider two rays of light that enter from above perpendicular to the plane of the lens and plate. One ray is reflected back upward at point A. The other ray passes from the lens into the air and is reflected back upward at point B. Both rays pass up through the
microscope eyepiece. These rays will interfere constructively if they are in phase, and destructively if they are out of phase.

The path difference for these two rays can be related to the radius of curvature of the lens using the figure below.

![Figure 2: Diagram Showing Geometry](image)

In this figure, $R$ is the radius of curvature of the lens, $r$ is the radius of one of the observed rings, and $2t$ is the path difference for the two rays.

From the right triangle,

$$R^2 = (R - t)^2 + r^2$$
$$R^2 = R^2 - 2Rt + t^2 + r^2$$
$$0 = -2Rt + t^2 + r^2$$
$$2Rt - t^2 = r^2$$

Since $t$ is on the order of $30\lambda \approx 20 \, \mu m$ and $R$ is on the order of 1 m, $2Rt$ is $10^5$ times larger than $t^2$. Therefore, $2Rt - t^2 \approx 2Rt$ and

$$t \approx \frac{r^2}{2R} = \frac{d^2}{8R}$$

In the last form, $d$ is the diameter of a ring. You will measure the diameters of the bright rings in this experiment.

If we are observing a bright ring from above the line AB, then we are seeing constructive interference. Because the light reflected upward at B undergoes one phase change and the light reflected up at A undergoes no phase change, our condition for constructive interference is

$$2t = (m + \frac{1}{2})\lambda,$$

where $m$ has an integer value. Since $t$ must be positive, $m$ can have values 0, 1, 2, …
We could directly combine the above equations and solve for $R$. We would, however, get a very inaccurate result. This is because the "point" of contact between the plane and convex surfaces is in fact an area of irregular shape. Furthermore, varying the force of contact between the surfaces varies the size of the area of contact, along with the sizes of all of the rings. For this reason differences in $r$ are less dependent on the force of contact between the surfaces than are individual values of $r$.

Consider the bright ring with $m = n$. For this ring we have the equations

$$t_n = \frac{d_n^2}{8R} \quad \text{and} \quad 2t_n = \left( n + \frac{1}{2} \right) \lambda .$$

Now, consider the bright ring with $m = n + k$. For this ring we have the equations

$$t_{n+k} = \frac{d_{n+k}^2}{8R} \quad \text{and} \quad 2t_{n+k} = \left( n + k + \frac{1}{2} \right) \lambda .$$

Subtracting we get the two equations,

$$t_{n+k} - t_n = \frac{d_{n+k}^2 - d_n^2}{8R} \quad \text{and} \quad 2t_{n+k} - 2t_n = k\lambda .$$

From these, we get $R = \frac{d_{n+k}^2 - d_n^2}{4k\lambda} .$

II. LABORATORY PROCEDURE

1. Shine white light on the stage of the traveling microscope. Adjust the eyepiece so that the crosshairs are in sharp focus. Rotate the barrel of the microscope so that one crosshair is perpendicular to the direction of travel of the carriage. Rotate the thumbwheel so that the index mark is near the center of the major scale, to avoid the danger of going off-scale later.

2. Hold the rings apparatus so that white light is reflected from it directly into your eye. Adjust the thumbscrews so that they exert a weak force on the apparatus, and so that the center of the ring pattern is near the center of the apparatus.

3. Place the rings apparatus, thumbscrews down, on the stage of the traveling microscope. Place the wire mesh on the glass surface. Slide the apparatus so that the scrap is directly under the objective lens of the microscope. Illumine the scrap and look for it in the microscope. Focus the microscope by moving the barrel up or down. Once you focus on the scrap, the microscope will be approximately focused for the interference rings. Remove the wire mesh.
4. Refer to Figure 1 in section IV. Place the microscope slide, tilted at about a 45° angle to the horizontal plane, between the rings apparatus and the objective lens of the microscope. Align the long dimension of the slide parallel to the direction of travel of the microscope. Plug in, turn on, and position the mercury lamp nearby and position it so that light reflects from the slide downward onto the rings apparatus. It will work best to place the lamp on its side with the green filter side forming a vertical plane as close to the microscope as possible.

5. Look through the microscope and locate the interference rings. If a bright glare fills the field of view, it may be due to light which reflects directly upward from the edge of the slide; move the slide toward the source to correct this condition. Center the ring pattern in the field of view. Focus the microscope for maximum sharpness of the rings. Adjust the angle of the microscope slide for maximum contrast between bright and dark rings.

6. Rotate the thumbwheel to move the carriage of the microscope until the perpendicular crosshair is on bright ring #33 on either side of the center. Before taking the position reading, move the carriage at least one tenth thumbwheel revolution past the ring, and then come back to it. (Errors due to thread backlash are eliminated if the carriage is always moved in the same direction just before taking a reading. If you pass a ring in further readings, repeat this process to eliminate backlash) Stop when the crosshair is tangent to the brightest part of the ring. Take the reading in millimeters, noting that the major scale gives millimeters, from 0 to 50 (labeled as 0 to 5 cm), and that the minor scale gives hundredths of millimeters, from 0 to 100. You should estimate the third decimal place in the measurement.

7. Proceed toward the center of the ring pattern, taking readings at rings 32, 31, 13, 12 and 11. If you go past any ring, back up at least one-tenth revolution, and approach it again.

8. Continue on past the center of the rings and take readings at rings 11, 12, 13, 31, 32 and 33 on the other side.

9. Rotate the rings apparatus approximately 90° about a vertical axis, and repeat steps 6-8.

III. CALCULATIONS

1. Using the data of II-6 to II-8, make a table containing the quantities n, diameter of nth ring, n+k, diameter of (n+k)th ring and radius of curvature of the lens. Let n take values of 11, 12, and 13. Let n+k take values of 31, 32, and 33. Then k is 20 in each case. Show the calculation of the radius of curvature of the lens in one case. Give all distances, including the wavelength of the light used, in mm. Find the average value of the radius of curvature of the lens from this part of the data. The radius of curvature should be somewhere between 1000 and 5000 mm.

2. Repeat step 1, using the data of II-9.

3. Find the percent difference between the average values of the radius of curvature found in step 1 and in step 2. A portion of the error is experimental another portion is due to the astigmatism of the lens. Astigmatism of a lens is the failure of the convex surface to be perfectly spherical.