Kinematics Problem Solving I

Frequently kinematics problems will be solved algebraically using the definitions and equations of motion for constant acceleration.

Definitions: $v_{av} \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$ $\qquad a_{av} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$

Equations of motion for constant acceleration:

$$(0) \quad x_f = x_i + \tfrac{1}{2}(v_i + v_f)t \qquad (2) \quad v_f = v_i + a\,t$$
$$(1) \quad x_f = x_i + v_i\,t + \tfrac{1}{2}a\,t^2 \qquad (3) \quad v_f^2 = v_i^2 + 2a\,(x_f - x_i)$$

Or

$$(0) \quad x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad (2) \quad v = v_0 + a\,t$$
$$(1) \quad x = x_0 + v_0\,t + \tfrac{1}{2}a\,t^2 \qquad (3) \quad v^2 = v_0^2 + 2a\,(x - x_0)$$

Sometimes it may be convenient to look at solutions (especially for problems involving more then one object in motion) either using freeze-frame diagrams (pictures of the objects at different states) or graphical interpretations.

We will use all three methods in the process of solving the problems given below and additional problems.

Kinematics problems may involve one or more objects. The object(s) in question may pass through one or several stages.

A. One Object, One Stage Problems:

Example A: You are driving a car at 30.0 mph when you begin to apply the brakes. Your car stops after travelling 152 ft. What acceleration (assumed constant) did the car have while you were applying the brakes?

1. A small airplane has a lift-off speed of 120 km/h.
   (a) What minimum constant acceleration does this require if the airplane is to lift off before reaching the end of a 240-meter runway?
   (b) How long does it take the airplane to become airborne?

2. A racing car reaches a speed of 40.0 m/s. At this instant, it begins a uniform negative acceleration, using a parachute as a braking system, and comes to rest in 5.00 s.
   (a) Determine the acceleration of the car.
   (b) How far does the car travel after acceleration starts?

3. A train 385 m long is moving on a straight track with a speed of 82.4 km/h. The engineer applies the brakes at a crossing, and later the last car passes the crossing with a speed of 16.4 km/h. Assuming constant acceleration, determine how long the train blocked the crossing. Disregard the width of the crossing.

4. A bullet is fired through a board 10.0 cm thick in such a way that the bullet's line of motion is perpendicular to the face of the board. The initial speed of the bullet (as it enters the board) is 24.0 km/min and it emerges from the other side of the board with a speed of 18.0 km/min. Find the acceleration of the bullet as it passes through the board and the total time the bullet is in contact with the board.

B. One Object, Multiple Stages Problems

Example B: A model rocket is launched from the ground. It starts from rest and accelerates upward at 15.0 m/s^2 for 3.00 s. Then, it has a downward acceleration of 9.80 m/s^2 until it reaches its highest point. What is the highest above the ground that the rocket gets?

1. A car is travelling at 72.0 km/h, when the driver notices an overturned truck blocking the road 95.0 m ahead. The car travels at 72.0 km/h for 0.50 s before the driver applies the brakes.
   (a) How far does the car travel in that 0.50 s?
   (b) If the car has an acceleration of -2.50 m/s^2, will it be able to stop before reaching the truck? If not, how fast is the car traveling when it collides with the truck?

2. A record of travel along a straight path is as follows: Start from rest with constant positive acceleration of 3.00 m/s^2 for 15.0 s; constant velocity for the next 2.00 min; constant negative acceleration of -9.50 m/s^2 for 4.40 s.
   (a) What was the total displacement for the complete trip?
   (b) What were the average speeds for legs 1, 2, and 3 of the trip as well as for the complete trip?

3. A car starts from rest and travels for 5.0 s with a uniform acceleration of +1.5 m/s^2. The driver then applies the brakes, causing a uniform acceleration of -2.0 m/s^2. If the brakes are applied for 3.0 s, how fast is the car going at the end of the braking period, and how far has it gone?

C. Two Objects

Example C: See Preparation for Problem Solving II, (9) (condor and cars). A California condor flying over the desert admires the beautiful hues and shading of the soft dusk light. She looks down and sees telephone poles on a straight, horizontal road. From experience, the condor knows that the distance between the poles is equal to the height of one pole. She spots two cars approaching each other with constant speeds. One car is at pole #1; the other is at pole #7. She notices that the car at pole #1 is traveling twice as fast as the car at pole #7. Where are the two cars when they pass each other?

1. Runner A is initially 4.0 mi west of a flagpole and is running with a constant velocity of 6.0 mi/h due east. Runner B is initially 3.0 mi east of the flagpole and is running with a constant velocity of 5.0 mi/h due west. How far are the runners

from the flagpole when their paths cross? Solve this problem using each of the three methods discussed. Do your answers agree?

2. Sheila leaves her home to walk to school at 7:30 am. After she leaves, her father notices she has forgotten her lunch. He walks outside and calls to her, but Sheila is listening to music through headphones and does not hear her father. Her father starts walking after her. He walks at 6ft/s. Sheila is walking at 4 ft/s. If Sheila is 60 ft in front of her father when he begins his pursuit, how far will her father walk before catching up to her?

3. A speeder passes a parked police car at 30.0 m/s. The police car starts from rest with a uniform acceleration of 2.44 m/s^2. The speeder's velocity is constant.
   (a) On a single set of axes sketch the positions of the speeder and the police car as functions of time.
   (b) How much time passes before the speeder is overtaken by the police car?
   (c) How far does the speeder get before being overtaken by the police car?

4. A hockey player is standing on her skates on a frozen pond when an opposing player, moving with a uniform speed of 12 m/s, skates by with the puck. After 3.0 s, the first player decides to chase her opponent. She maintains an acceleration of 4.0 m/s^2.
   (a) Sketch the position of each player as a function of time.
   (b) How long does it take the first player to catch her opponent?
   (c) How far has she traveled in this time?
   Assume the player with the puck remains in motion at constant speed.