RATIONAL EXPRESSIONS AND EQUATIONS

ADDING RATIONAL EXPRESSIONS

In the following examples, we are finding a common denominator and then adding like fractions.

1. $\frac{2x}{3}+\frac{1}{5x}$

LCD: 15x since 3 * 5x = 15x.

$\frac{2x}{3}\bullet\frac{5x}{5x}+\frac{1}{5x}\bullet\frac{3}{3}$

Multiply each fraction by missing part of denominator over itself to get a common denominator.

$\frac{10x^2}{15x}+\frac{3}{15x}$

Multiply numerators and denominators of each fraction.

$\frac{10x^2+3}{15x}$

Add numerators. Keep the same denominator the same way we add like fractions.

2. $\frac{2x}{x^2-9}+\frac{1}{x-3}$

LCD: (x-3)(x+3) – Factored form of $x^2 – 9$.

$\frac{2x}{x^2-9}+\frac{(x+3)}{(x+3)}\left(\frac{1}{x-3}\right)$

Multiply each fraction by missing part of denominator over itself (only need to do this on right fraction here) to get a common denominator.

$\frac{2x}{x^2-9}+\frac{x+3}{x^2-9}$

Multiply numerators and denominators of each fraction as needed.

$\frac{3x+3}{x^2-9}$

Add numerators, Keep the same denominator. the same way we add like fractions.

SIMPLIFYING COMPLEX RATIONAL EXPRESSIONS

In the next group of examples we will be multiplying all terms by the LCD of all of them.

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3. $\dfrac{\frac{2x}{3}+\frac{1}{5x}}{\frac{2}{x}+\frac{9}{15}}$

LCD: 15x (This is the LCD of all 4 fractions since 3 * 5x = 15x

$\dfrac{15x\left(\frac{2x}{3}+\frac{1}{5x}\right)}{15x\left(\frac{2}{x}+\frac{9}{15}\right)}$

Multiply numerator and denominator by LCD (15x).
We are really multiplying by 1 since $\frac{15x}{15x}$ = 1.

$\dfrac{15x\left(\frac{2x}{3}\right)+15x\left(\frac{1}{5x}\right)}{15x\left(\frac{2}{x}\right)+15x\left(\frac{9}{15}\right)}$

Distribute 15x over each fraction so we can cancel the denominators in the next step.

$\dfrac{5x(2x)+3x(1)}{15(2)+x(9)}$

Simplify by cancelling individual denominators.

$\dfrac{10x^2+3}{30+135x}$

Multiply terms in numerator and denominator.

4. $\dfrac{1-\frac{9}{x^2}}{1-\frac{2}{x}-\frac{3}{x^2}}$

LCD: x^2
This is the LCD of all five terms since the denominator of 1 is 1.

$\dfrac{x^2\left(1-\frac{9}{x^2}\right)}{x^2\left(1-\frac{2}{x}-\frac{3}{x^2}\right)}$

Multiply numerator and denominator by x^2.
We are actually multiplying by one since $\frac{x^2}{x^2}$ = 1.

$\dfrac{x^2(1)-x^2\left(\frac{9}{x^2}\right)}{x^2(1)-x^2\left(\frac{2}{x}\right)-x^2\left(\frac{3}{x^2}\right)}$

Distribute x^2 over all five terms so we can cancel the denominators in the next step.

$\dfrac{x^2(1)-1(9)}{x^2(1)-x(2)-1(3)}$

Simplify by cancelling individual denominators where needed.

$\dfrac{x^2-9}{x^2-2x-3}$

Multiply terms in numerator and denominator.

$\dfrac{(x-3)(x+3)}{(x-3)(x+1)}$

Factor numerator and denominator to simplify further.

$\dfrac{x+3}{x+1}$

Cancel $\frac{x-3}{x-3}$ since this equals 1.

RATIONAL EQUATIONS

Just as in simplifying complex fractions, when solving rational equations, we multiply all terms by the LCD of all of them.

5. $\frac{x}{15}+\frac{1}{3x}=\frac{2}{5}$

LCD = 15x since 3x * 5 = 15x.

$15x\left(\frac{x}{15}+\frac{1}{3x}\right)=15x\left(\frac{2}{5}\right)$

Multiply both sides of the equation by 15x. Whatever we do to one side of the equation, we must do the same to the other side of the equal sign. We do this to cancel the denominators soon,

$15x\left(\frac{x}{15}\right)+15x\left(\frac{1}{3x}\right)=15x\left(\frac{2}{5}\right)$

Distribute 15x over each of the terms on the left so we can cancel the denominators in the next step.

$x^2 + 5 = 6x$

Simplify each term by cancelling individual denominators.

$x^2 + 5 – 6x = 6x – 6x$

Subtract 6x from both sides of the equation to set quadratic equation = 0.

$x^2 – 6x + 5 = 0$

Combine like terms.

$(x – 5)(x – 1) = 0$

Factor.

$X = 5$ or $x = 1$

Solve using the zero product property. One of the factors must equal 0.

6. $1-\frac{2}{x}-\frac{3}{x^2}=\frac{-4}{x^2}$

LCD: x^2

$x^2\left(1-\frac{2}{x}-\frac{3}{x^2}\right)=x^2\left(\frac{-4}{x^2}\right)$

Multiply both sides of the equation by x^2. Whatever we do to one side of the equation, we must do the same to the other side of the equal sign. We do this to cancel the denominators soon,

$x^2(1)-x^2\left(\frac{2}{x}\right)-x^2\left(\frac{3}{x^2}\right)=x^2\left(\frac{-4}{x^2}\right)$

Distribute x^2 over each of the terms on the left so we can cancel the denominators in the next step.

$x^2(1)-x(2)-3=-4$

Simplify by cancelling individual denominators where needed.

$X^2 – 2x – 3 = -4$

Multiply terms as needed.

$x^2 – 2x + 1 = 0$

Add 4 to both sides of the equation to get a quadratic equation we can factor in the next step.

$(x – 1)(x – 1) = 0$

Factor so we can solve for x.

$x – 1 = 0$

Zero product property, one factor must equal 0.

$x = 1$

Solve for x.