Completing the Square – One method for solving Quadratic Equations

$3x^2 + 7x - 2 = 0$

$3x^2 + 7x = 2$

$3(x^2 + \frac{7}{3}x) = 2$

$x^2 + \frac{7}{3}x = \frac{2}{3}$

$Think\left(\frac{1}{2}\cdot\frac{7}{3}\right)^2 = \left(\frac{7}{6}\right)^2 = \frac{49}{36}$

$x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 = \frac{2}{3} + \left(\frac{7}{6}\right)^2$

$\left(x + \frac{7}{6}\right)^2 = \frac{2}{3} + \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{24}{36} + \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{73}{36}$

$\sqrt{\left(x + \frac{7}{6}\right)^2} = \pm\sqrt{\frac{73}{36}}$

$x + \frac{7}{6} = \pm\sqrt{\frac{73}{36}}$

$x = -\frac{7}{6} \pm \sqrt{\frac{73}{36}}$

$x = -\frac{7}{6} \pm \frac{\sqrt{73}}{6}$

1. Move the constant to the right side of the equation.
2. Factor out the leading coefficient.
3. Divide both sides by the leading coefficient.
4. Square ½ of the coefficient of x.
5. Add this value to both sides of the equation.
6. Factor the left side as a binomial square
7. Simplify the right side.
8. Take the square root of both sides of the equation.
9. Simplify
10. Solve for x.

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

$a(x^2 + \frac{b}{a}x) = -c$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$Think\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b^2}{4a^2}\right)$

$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \left(\frac{b^2}{4a^2}\right)$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{-4ac + b^2}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{-4ac + b^2}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$