

Figure 1 shows a bottom view of the spherometer.

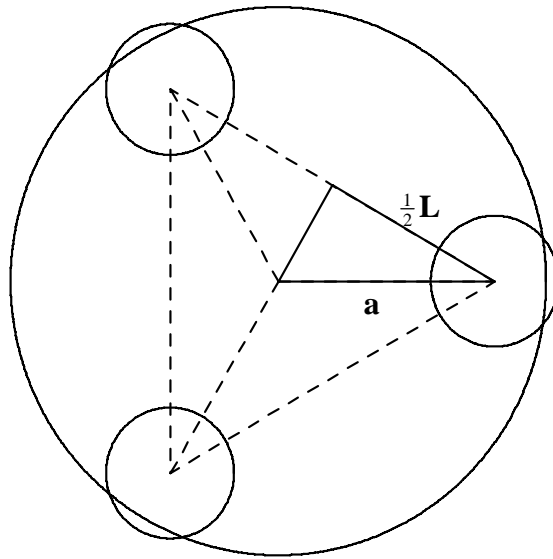


Figure 1

Let  $L$  equal the center-to-center separation of the feet.  $L$  is the difference between the outer-to-outer separation of the feet and the diameter of the feet, both of which are measured.

Let  $a$  be the distance from the center of each foot to the center of the system  $P$ .

In the right triangle shown in the figure,  $\frac{1}{2}L = a \cos(30^\circ) = \frac{\sqrt{3}}{2}a$ . So,  $a = \frac{1}{\sqrt{3}}L$ .

Figure 2 shows one foot of the spherometer in contact with a sphere of unknown radius.

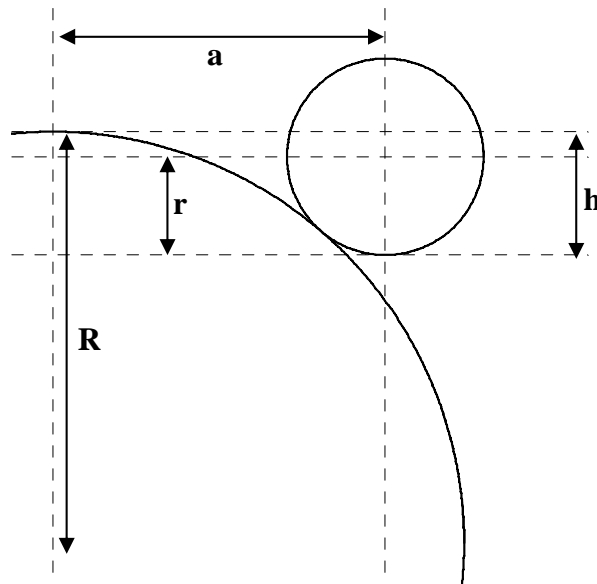


Figure 2

In this figure,  $R$  is the unknown radius of the sphere,  $r$  is the radius of the spherometer foot, and  $h$  is the difference between the spherometer reading when in contact with the sphere and the reading when in contact with a flat surface.

Figure 3 shows a right triangle superimposed on Figure 2.

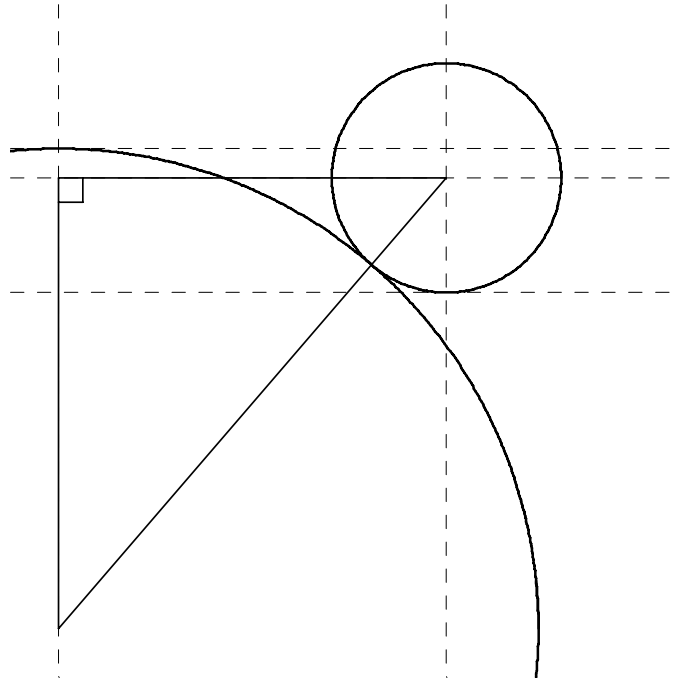


Figure 3

With the help of Figure 2, it is found that the length of the hypotenuse of the right triangle is  $\mathbf{R + r}$ . The lengths of the two legs are  $\mathbf{R - h + r}$  and  $\mathbf{a}$ .

Applying the Pythagorean Theorem,  $(\mathbf{R + r})^2 = ((\mathbf{R + r}) - \mathbf{h})^2 + \mathbf{a}^2$ .

$$(\mathbf{R + r})^2 = (\mathbf{R + r})^2 - 2\mathbf{h}(\mathbf{R + r}) + \mathbf{h}^2 + \mathbf{a}^2$$

$$0 = -2\mathbf{h}(\mathbf{R + r}) + \mathbf{h}^2 + \mathbf{a}^2$$

$$2\mathbf{h}(\mathbf{R + r}) = \mathbf{h}^2 + \mathbf{a}^2$$

$$\mathbf{R + r} = \frac{\mathbf{h}}{2} + \frac{\mathbf{a}^2}{2\mathbf{h}}$$

$$\mathbf{R} = \frac{\mathbf{h}}{2} + \frac{\mathbf{a}^2}{2\mathbf{h}} - \mathbf{r}$$

Replacing  $\mathbf{a}^2$  with  $\frac{1}{3}\mathbf{L}^2$  gives our final result:  $\mathbf{R} = \frac{\mathbf{h}}{2} + \frac{\mathbf{L}^2}{6\mathbf{h}} - \mathbf{r}$ .