EXPERIMENT 16
THE MAGNETIC MOMENT OF A BAR MAGNET AND THE HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FIELD

I. THEORY

The purpose of this experiment is to measure the magnetic moment μ of a bar magnet and the horizontal component B_E of the earth's magnetic field.

Since there are two unknown quantities, μ and B_E, we need two independent equations containing the two unknowns. We will carry out two separate procedures: The first procedure will yield the ratio of the two unknowns; the second will yield the product. We will then solve the two equations simultaneously.

The pole strength of a bar magnet may be determined by measuring the force F exerted on one pole of the magnet by an external magnetic field B_0. The pole strength is then defined by

$$p = F/B_0$$

Note the similarity between this equation and q = F/E for electric charges.

In Experiment 3 we learned that the magnitude of the magnetic field, B, due to a single magnetic pole varies as the inverse square of the distance from the pole.

$$B = \frac{k' p}{r^2}$$

in which k' is defined to be 10^{-7} N/A^2.

Consider a bar magnet with poles a distance 2x apart. Consider also a point P, located a distance r from the center of the magnet, along a straight line which passes from the center of the magnet through the North pole. Assume that r is much larger than x. The resultant magnetic field B_m at P due to the magnet is the vector sum of a field B_N directed away from the North pole, and a field B_S directed toward the South pole. The distances from P to the North and South poles are r - x and r + x, respectively. The magnitude of the resultant field at P is

$$B_m = B_N - B_S = \frac{k' p}{(r - x)^2} - \frac{k' p}{(r + x)^2}$$

Putting the two terms over a common denominator, we obtain

$$B_m = \frac{4xrk' p}{\left(r^2 - x^2\right)^2}$$

Since, x is small compared to r, we can neglect the x^2 in the denominator.

$$B_m = \frac{4xrk'p}{r^4} = \frac{4xk'p}{r^3}$$

This can be written in terms of the magnetic moment of the bar magnet, μ defined by the formula $\mu \equiv 2px$.

$$B_m = \frac{2k'\mu}{r^3}.$$

Note the similarity between the definition of the magnetic moment and the equation for the field strength to the definition of the electric dipole moment ($p \equiv qd$) and the equation for the field strength of an electric dipole a distance r along the axis of the dipole ($E = \frac{2kp}{r^3}$). In these equations, d is the separation of the charges and k is Coulomb's constant.

Our first experimental procedure will yield the ratio of μ to B_E. We will do this indirectly by comparing B_m to B_E, using a magnetometer. The magnetometer, which was also used in Experiment 3, consists of a small magnetized disk attached to a long non-magnetic pointer, pivoted on a vertical axis. The pointer is mounted at right angles to the direction of magnetization of the disk.

With the bar magnet far from the magnetometer, the only significant magnetic field acting on the magnetized disk will be the horizontal component B_E of the earth's field. (The vertical component of the earth's field has no effect on the disk, because the disk cannot rotate about a horizontal axis.) In this case the North pole of the magnetized disk will point toward magnetic north, and the non-magnetic pointer will point in the magnetic east-west direction. When the housing is properly oriented, the pointer will read zero, and a meter stick, attached to the housing, will be oriented in the magnetic east-west direction.

If the bar magnet is placed on the meter stick with its North pole toward the magnetic east, a field B_m, directed toward the magnetic east, will also act on the magnetized disk.

The resultant field at the center of the magnetometer is the vector sum of $\mathbf{B}_e$ toward the magnetic north and $\mathbf{B}_m$ toward the magnetic east. Let θ be the angle between $\mathbf{B}_e$ and the resultant field $\mathbf{B}$. The disk and pointer must now rotate clockwise through the angle θ until the North pole of the disk points in the direction of the resultant field. Since the pointer originally read zero, it will now read θ. A simple diagram shows that

$$B_m = B_E \tan\theta$$

We now have an equation that will give us the ratio of μ to B_E.

$$B_E = \frac{2k'\mu}{r^3}\cot\theta$$

For this experiment, we will measure the angle θ for several different values of r. The ratio of μ to B_E can be calculated from the slope of a graph of cot θ versus r^3.

The product of μ and B_E may be found by suspending the bar magnet from a nearly torsion-free string, giving it a small angular displacement from equilibrium, and allowing it to oscillate in simple harmonic motion. The earth's horizontal field B_E provides the restoring torque.

The torque exerted by a uniform magnetic field B_E on a bar magnet of magnetic moment μ is given by

$$\tau = \boldsymbol{\mu} \times \mathbf{B}_E \text{ or } \tau = \mu B_E \sin\phi$$

in which ϕ is the angle between μ and B_E, which is the same as the angle between the instantaneous position of the bar magnet and its equilibrium position. (The vertical component of the earth's magnetic field has no effect in this part of the experiment either, because the bar magnet is not free to rotate about a horizontal axis.)

If the angle ϕ is small, then sin ϕ can be approximated as ϕ and the restoring torque will be proportional to ϕ. If the magnet has a moment of inertia, I, the angular acceleration is related to this torque by

$$I\alpha = -\mu B_E \phi \quad \text{or} \quad \alpha = -\frac{\mu B_E}{I}\phi \ .$$

This equation describes simple harmonic motion with angular frequency $\omega = \sqrt{\frac{\mu B_E}{I}}$ and period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\mu B_E}}$.

II. LABORATORY PROCEDURE

CAUTION: If you have a mechanical watch, keep it at some distance from the bar magnet at all times. Use care not to drop the magnet. Jolts tend to demagnetize any magnet, reducing its magnetic moment, which is one of the unknown quantities we are measuring.

1. Place the magnetometer on top of an inverted wooden box to remove it from ferrous metal in the laboratory table. Do not tilt the magnetometer excessively, or the glass plate (on some models) may fall out and break. Place the level on top of the glass plate of the magnetometer and level the instrument. Remove the level some distance from the magnetometer.

2. Remove the bar magnet some distance from the magnetometer. Also remove any ferrous metals, such as the level and certain mechanical pencils. Turn the knob or wheel to raise the pivot until the pointer moves freely; however, do not raise the pivot so high that the pointer is pushed against the glass plate. Rotate the housing so that both ends of the pointer read zero. If the pointer is somewhat bent, both ends should be off from zero by equal amounts.

3. If the magnetometer lacks a built-in meter stick, insert a ***thin*** meter stick into the brackets, and center it.

4. Place the bar magnet on the meter stick so that its center is 20.0 cm from the center of the magnetometer. This is accomplished most easily by placing the two ends of the magnet equally distant from the 20.0 cm mark. The magnet must be parallel to the meter stick. Read and record both ends of the pointer, estimating to the nearest tenth of a degree. Treat all angles as positive. Record the data of steps 4-6 in tabular form.

5. Repeat step 4 for distances of 22, 24, 26, 28 and 30 cm.

6. Repeat steps 4 and 5 with the bar magnet reversed in direction.

7. Remove the magnetometer a considerable distance from the box. Set up a table clamp, a vertical rod, a right angle clamp, and a horizontal rod, with the horizontal rod above the box. Obtain two strings, one long and one short, with loops at each end. Pass one loop of the long string over the horizontal rod. Pass the short string through the other loop of the long string. Pass the bar magnet through both loops of the short string. Adjust the rods and clamps so that the bar magnet is suspended in a horizontal plane at the same position as was previously occupied by the center of the magnetometer. (The same location is used because the Earth's magnetic field is not uniform throughout the laboratory room.)

8. Twist the bar magnet 10 or 20° from its equilibrium position, and release it. It should oscillate in angular simple harmonic motion, twisting the string (not simple pendulum motion). Use a clock or watch to time 20 cycles. (During one cycle the magnet moves from one side to the other and back again.) It is sufficient to estimate time to the nearest second. Repeat and average. If the two times differ by more than 2%, repeat the timings.

9. Use one of the strings to suspend the bar magnet above the pan of the balance. Measure the mass to at least three significant figures. ***Do not magnetize the pan by placing the magnet directly on it. Do not discard the strings.***

10. Use a vernier caliper to measure the length of the magnet.

11. If a meter stick was attached to the magnetometer in step 3, remove it.

III. CALCULATIONS

1. Calculate the moment of inertia of the magnet.

2. List the following quantities in tabular form: r (in meters), r^3, θ, and cot θ. θ must be the average of the **four** values measured at each distance.

3. Plot a graph of cot θ versus r^3. Include the origin on the graph. Use a straightedge to draw the straight line that best fits the plotted points, and also passes through the origin. Does cot θ appear to be directly proportional to r^3?

4. Use the Modified Least Squares formula to calculate the slope of the graph. The formula can be found in the Introduction of this Lab Manual.

5. From the slope, calculate the ratio of the magnetic moment of the bar magnet to the horizontal component of the Earth's magnetic field.

6. Calculate the average experimental period of the bar magnet when oscillating in simple harmonic motion. Remember, the period is the time for one cycle. Calculate the product of the magnetic moment of the bar magnet and the horizontal component of the Earth's magnetic field.

7. Calculate the magnetic moment of the bar magnet.

8. Calculate the horizontal component of the earth's magnetic field at the location of the apparatus. Convert the result to μT.