

EXPERIMENT 12 THE RC SERIES CIRCUIT

I. THEORY

In this experiment we will charge and discharge a capacitor through a resistor, and measure the voltage across the capacitor as a function of time. The purpose of this experiment is to test the theoretical equations which describe this process, and also to measure the time constant RC of the resistor-capacitor combination.

When an uncharged capacitor C is connected in series with a resistor R , and then this combination is connected to a cell of emf V_0 and negligible internal resistance, the voltage across the capacitor, as a function of time, is given by the equation

$$V_c = V_o \left(1 - e^{-t/RC} \right) \quad (1)$$

The subscript c stands for charging of the capacitor. The product RC is called the time constant of the circuit and, for this circuit, is equal to the amount of time required for the voltage of the capacitor to reach 63.2 % of its ultimate value V_o . The symbol τ is sometimes used to represent a time constant. For a series RC circuit $\tau = RC$.

If a capacitor, which has been charged to a voltage V_o , is allowed to discharge through a resistor R , the voltage across the capacitor, as a function of time, is given by the equation

$$V_d = V_o e^{-t/RC} \quad (2)$$

The subscript d stands for discharging of the capacitor. The time constant, RC , for this circuit is the time required for the voltage of the capacitor to fall to 36.8 % of its initial value V_o .

If equations (1) and (2) are plotted on linear graph paper, curved lines result in both cases. If they are plotted on semi logarithmic paper, with time on the linear axis, equation (1) still yields a curved line, but equation (2) yields a straight line.

For greater precision in determining the experimental time constant, it is desirable to express equations (1) and (2) in different forms, so as to yield linear graphs when using linear graph paper.

Equation (1) may be written in the form

$$\frac{V_o - V_c}{V_o} = e^{-t/RC} \quad (3)$$

Taking natural logarithms, we obtain

$$\ln\left(\frac{V_o - V_c}{V_o}\right) = -t/RC \quad (4)$$

From this equation we see that if we plot the natural logarithm of $(V_o - V_c)/V_o$ on the vertical axis of linear graph paper, and time on the horizontal axis, we obtain a straight line of slope $-1/RC$.

Equation (2) may be written in the form

$$V_o/V_d = e^{t/RC} \quad (5)$$

Taking natural logarithms, we obtain

$$\ln\left(V_o/V_d\right) = t/RC \quad (6)$$

From this equation we see that if we plot the natural logarithm of (V_o/V_d) on the vertical axis of linear graph paper, and t on the horizontal axis, we obtain a straight line of slope $1/RC$.

An ordinary voltmeter cannot be used to read the voltage across a capacitor, since such a meter would discharge the capacitor very quickly. Instead, we will use an electrometer (EM) having very high resistance. Since the EM scale is rather small, we will connect the EM output (designed for a computer interface) to an ordinary voltmeter (VM). To avoid inputting large voltages into a computer-interface, the EM is designed such that its output is one-twentieth of the measured voltage. We will multiply the voltmeter readings by 20 to get the voltage measured by the electrometer.

II. PROCEDURES

1. With the portable power supply (PPS) turned off and the VOLTAGE and CURRENT control knobs in the full counterclockwise positions, wire up the circuit shown in the diagram in section IV. It is recommended that the components be laid out on the laboratory table in the same relative positions as in the circuit diagram.

- Use spade terminal to spade terminal wires to connect the capacitor to the resistor and to the switch, as shown in the diagram.
- Run another spade terminal to spade terminal wire from the resistor to the switch, as shown. When making connections to the switch S, be sure to use the three terminals ON THE SAME SIDE so as to have, in effect, an SPDT switch.

- Use banana plug-spade terminal wires to connect the portable power supply (PPS) to the switch and to the resistor. Use only the positive and negative banana jacks of the PPS.
- Use the BNC to alligator clips on the input cable in the plastic bag in the equipment box to connect the electrometer (EM) to the rest of the circuit. Use the banana-to-banana wires provided to ground the electrometer to any electrical wall outlet.
- Connect the output of the EM to the multimeter (which will be used as a voltmeter (VM) in this lab), using the special cable provided. Use the two input jacks on the right side of the VM. The negative (black) banana plug of the cable should be connected to the COM jack on the VM and the red to the “V Ω ”.

2. Record the nominal values of the resistor and capacitor. MFD means microfarads, though M is not the correct metric prefix for micro-

3. Connect the 10-k Ω shorting lead (a resistor with an alligator clip connected to each lead) in parallel with R. Turn the electrometer on and set it on the 30 V range. Turn on the multimeter by turning the dial to the 2 V setting. (During lab, you will record values from the multimeter. Before doing calculations these will need to be multiplied by a factor of 20.) Push the zero button to zero the electrometer. (Note: If the electrometer cannot be zeroed, disconnect the output cable and reconnect and then try to zero it. If this does not work, please see the instructor. It may be necessary to record a zero correction for the VM readings.)

4. Leave the VOLTAGE control knob in the minimum position, the far counterclockwise (CCW) position. Turn the CURRENT control knob to the maximum position, the far CW position. Turn the PPS on and adjust the VOLTAGE control knob until the portable power supply reads 20 V. Record the voltage reading on the PPS. Leave the PPS at this setting.

5. Throw S to the neutral position. Remove the 10-k Ω shorting lead from the resistor and hold it in contact with the terminals of the capacitor for about 5 seconds, to discharge it. The EM and VM readings should drop to zero, and remain there after the shorting lead is removed. If necessary, re-zero the EM.

6. CHARGING: While looking at a watch or clock, one partner should throw S to the CHG position at zero time, and give an audible signal every 5 seconds thereafter. Another partner should observe and record the VM readings at the end of every five-second interval, until a total of nine readings have been recorded, including the zero reading at zero time. During this time, no one should touch bare wires, as that could significantly change the data. The uncertainty in the voltage reading of the multimeter is in the third digit displayed. Be sure to record to the correct number of significant figures.

7. With S still in the CHG position, connect the 10-k Ω shorting lead in parallel with R for about 5 seconds, in order to charge the capacitor to 20V. Remove the shorting lead.

8. DISCHARGING: Record the voltmeter reading as the reading at time zero. Throw S to the DIS position and record the VM readings at five-second intervals until a total of nine readings have been recorded, including the reading at time zero.

9. Turn the EM off. Turn the VM off. Turn the voltage and current control knobs of the PPS to the far CCW position. Turn the PPS off.

III. CALCULATIONS

1. Make a table containing the quantities time, V_c , $(V_o - V_c)/V_o$, $\ln((V_o - V_c)/V_o)$, V_d , V_o/V_d , $\ln(V_o/V_d)$. V_o is 20 V. Voltages recorded in this table should be the 20 times the VM reading AFTER the zero correction has been applied.
2. Plot V_c and V_d versus time on linear graph paper. Use the same axes so that the curves will cross. Draw *smooth curves* through both sets of points. Label the curves "charging capacitor" and "discharging capacitor".
3. Calculate $0.632 V_o$. Make a mark on the V_c graph at this voltage. Read the corresponding value of time from the graph. This is an experimental value of the time constant.
4. Calculate $0.368 V_o$. Make a mark on the V_d graph at this voltage. Read the corresponding value of time from the graph. This is a second experimental value for the time constant.
5. Average the experimental time constants found in the two preceding steps. Calculate the per cent difference between this value and the nominal time constant RC .
6. Determine the theoretical time at which V_c should equal V_d . This is the time at which the curves on your graph should intersect. Calculate this time numerically, using the nominal values of R and C . Read from the graph the actual value of time at which they intersect. Find the percent difference between the two values of time.
7. Plot V_c and V_d versus time (linear axis) on two-cycle semi-logarithmic graph paper. Plot voltages from 1 V to 10 V on the lower cycle and voltages from 10 V to 30 V on the upper cycle. Zero volts cannot be plotted on this graph. Draw a straight line through the V_d values, and a smooth curve through the V_c values.
8. Using equations (1) and (2), obtain expressions for $\ln V_c$ and $\ln V_d$. Keeping in mind that a logarithmic axis on graph paper has the effect of taking the logarithm of the plotted quantity, explain why one curve of step 7 is linear, but the other is nonlinear.
9. Plot $\ln((V_o - V_c)/V_o)$ and $\ln(V_o/V_d)$ versus time (horizontal axis) on separate sheets of linear graph paper. On each graph, draw in the best fit straight line.

10. Determine the slope of each line. Average the absolute value of the slopes and use the average to determine an experimental value of the time constant. Find the percent difference between the experimental value of the time constant and the nominal value of the time constant RC .

IV. DIAGRAM

