EXPERIMENT 6
CHARGE SHARING BY CAPACITORS

I. THEORY

The purpose of this experiment is to test the theoretical equations governing charge sharing by capacitors and to measure the capacitance of an "unknown" capacitor.

Capacitance is defined by the equation

$$C \equiv Q/V \tag{1}$$

The unit of capacitance, the farad (F), is defined as one coulomb per volt.

When **two** capacitors are connected in series AND ARE INITIALLY UNCHARGED, the charges which they acquire subsequently must be equal, and the equivalent capacitance is given by the equation

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \tag{2}$$

When two capacitors are connected in parallel, their voltages must be equal, and their equivalent capacitance is given by the equation

$$C_{eq} = C_1 + C_2 \tag{3}$$

Kirchhoff's Second Rule states that the algebraic sum of the changes in potential around any closed path is zero, due to the uniqueness of potential at a point. We may write this in the form

$$\sum \Delta V = 0 \tag{4}$$

An ordinary voltmeter cannot be used to read the voltage across a capacitor, since such a meter would discharge the capacitor very quickly. Instead, we will use an electrometer (EM) having very high resistance. Since the EM scale is rather small, we will connect the EM output (designed for a computer interface) to an ordinary voltmeter (VM). To avoid inputting large voltages into a computer-interface, the EM is designed such that its output is one-twentieth of the measured voltage. We will multiply the voltmeter readings by 20 to get the voltage measured by the electrometer.

In circuit #1, in section IV, the switch S_1 is thrown to position 1 and C_1 charges up to a voltage V_1, the voltage of the power supply. Then S_1 is thrown to position 2 and C_1 shares its charge with C_2. This process is repeated several times. Although C_2 is initially uncharged in the FIRST cycle, this is not the case in subsequent cycles.

The resistors shown in this and other circuits serve only to protect the components from damage; they do not affect the final charges or voltages.

Let Q_1, Q_2, V_1 and V_2 be the respective charges and voltages of C_1 and C_2 just before throwing S_1 to position 2. Let Q be the total charge to be shared. Let V' be the common voltage after the charge is shared. By (1) we have

$$Q_1 = C_1V_1 \text{ and } Q_2 = C_2V_2 \tag{5}$$

The total charge Q is given by the equation

$$Q = Q_1 + Q_2 = C_1V_1 + C_2V_2 \tag{6}$$

Using equations (1), (3) and (6) we obtain

$$V' = \frac{Q}{C_{eq}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \tag{7}$$

Solving for C_1 we obtain

$$C_1 = C_2 \times \frac{V' - V_2}{V_1 - V'} \tag{8}$$

Thus, we may determine the value of an "unknown" capacitor C_1 in terms of a "known" capacitor C_2 and measured voltages V_1, V_2 and V'.

In circuit #2, in section IV, we have two capacitors and a power supply connected in series. Circuit #3 is used for charging and reading voltages of the capacitors of circuit #2. Resistors are used in both circuits to protect the components from damage. They do not affect the charges of the capacitors.

Suppose that the capacitors of circuit #2 are initially charged independently to voltages V_3 and V_4, where $(V_3 + V_4)$ is greater than the power supply voltage V_p. While S_2 is open, there must be a voltage across it, in order to satisfy Kirchhoff's Second Rule. When S_2 is closed, there can be no voltage across it, but Kirchhoff's Second Rule must still hold. The capacitors discharge partially in order to satisfy this.

Let the initial charges (before closing S_2) of C_3 and C_4 be Q_3 and Q_4. Let the final charges (after closing S_2) be Q_3' and Q_4'. Let the final voltages be V_3' and V_4'. Using equation (1) we obtain the four equations

$$Q_3 = C_3V_3,\ Q_4 = C_4V_4,\ Q_3' = C_3V_3',\ \text{and } Q_4 = C_4V_4' \tag{9}$$

Equation (2) cannot be used in analyzing this circuit, since the capacitors are charged unequally. Kirchhoff's Second Rule, on the other hand, is always valid. Applying Kirchhoff's Second Rule to circuit #2 we obtain

$$-V_3' - V_4' + V_p = 0 \tag{10}$$

Combining equations (9) and (10) we obtain

$$-Q_3'/C_3 - Q_4'/C_4 + V_p = 0 \tag{11}$$

Let q be the charge lost by each capacitor when S_2 is closed. Then the final charges are related to the initial charges by the equations

$$Q_3' = Q_3 - q \quad \text{and} \quad Q_4' = Q_4 - q \tag{12}$$

Combining equations (11) and (12) we obtain

$$-(Q_3 - q)/C_3 - (Q_4 - q)/C_4 + V_p = 0 \tag{13}$$

Solving for q we obtain

$$q = (C_3Q_4 + C_4Q_3 - C_3C_4V_p)/(C_3 + C_4) \tag{14}$$

Combining equations (9) and (14) we obtain

$$q = C_3C_4(V_3 + V_4 - V_p)/(C_3 + C_4) \tag{15}$$

Combining equations (9) and (12) we obtain the equations

$$V_3' = Q_3'/C_3 = (Q_3 - q)/C_3 = (C_3V_3 - q)/C_3 = V_3 - q/C_3 \tag{16}$$

$$V_4' = Q_4'/C_4 = (Q_4 - q)/C_4 = (C_4V_4 - q)/C_4 = V_4 - q/C_4 \tag{17}$$

Finally, combining equation (15) with equations (16) and (17), we obtain the equations

$$V_3' = (C_3V_3 + C_4V_p - C_4V_4)/(C_3 + C_4) \tag{18}$$

$$V_4' = (C_4V_4 + C_3V_p - C_3V_3)/(C_3 + C_4) \tag{19}$$

In part of this experiment, V_p will exceed $(V_3 + V_4)$. In such a case the capacitors become charged to higher voltages when S_2 is closed in circuit #2. The quantity q in equation (12) then becomes a negative number, but equations (18) and (19) are still valid.

II. LABORATORY PROCEDURE

1. With the portable power supply (PPS) turned off and the VOLTAGE and CURRENT control knobs in the full counterclockwise positions, wire up circuit #1, using the "unknown" capacitor for C_1 and the 500 nF (0.500 μF) standard capacitor for C_2. The 1000 nF (1.00 μF) standard capacitor will not be used in circuit #1. It is recommended

that the components be placed on the laboratory table in the same relative positions as in the circuit diagram before beginning to connect wires. Use mounted 10 kΩ resistors for R_1 and R_2. All connections will be made with spade terminal to spade terminal wires unless otherwise noted. For some connections, two wires will be attached to the same clip (eg, the side of R_2 connected to S_1 and C_2.

- Use the banana plugs of two banana plug to spade terminal leads to make connections to the positive and negative terminals of the PPS.
- Use the three binding posts on ONE SIDE of a double-pole-double-throw (DPDT) switch for S_1 and put it in the neutral position.
- Use the BNC to alligator clips on the input cable in the plastic bag in the equipment box to connect the electrometer (EM) to the rest of the circuit. Use the banana-to-banana wires provided to ground the electrometer to any electrical wall outlet.
- Connect the output of the EM to the multimeter (which will be used as a voltmeter, VM, in this lab) using the special cable provided. Use the two input jacks on the right side of the VM. The negative (black) banana plug of the cable should be connected to the COM jack on the VM.
- Record the nominal value of the "unknown" capacitance.

2. Connect the resistive shorting lead (a wire with a resistor in the middle and alligator clips on the ends) between points 1 and 2 of S_1. Turn the electrometer on and set it on the 10 V range. Turn on the multimeter by turning the dial to the 2 V setting. During lab, you will record values from the multimeter. Before doing calculations these will need to multiplied by a factor of 20. Push the zero button to zero the electrometer. (Note: If the electrometer cannot be zeroed, disconnect the output cable and reconnect and then try to zero it. If this does not work, please see the instructor.)

3. Leave the VOLTAGE control knob in the minimum position, the far counterclockwise (CCW) position. Turn the CURRENT control knob to the maximum position, the far CW position. Turn the PPS on and adjust the VOLTAGE control knob until the portable power supply reads 20 V. Record the voltage reading on the PPS. Leave the PPS at this setting.

4. Remove the resistive shorting lead from S_1. Set S_1 to position 2. Discharge the capacitors by holding the two ends of the shorting lead in contact with the terminals of either capacitor for about 5 seconds.

5. Throw S_1 to position 1. After about 2 seconds, throw it to position 2. Read and record the value of V' from the VM. Do not touch any part of the circuit except the switch, as your body may discharge the capacitors. In order to minimize charge leakage, proceed at once to step #6.

6. Repeat step #5 three more times. Do not discharge either capacitor between cycles. Turn the voltage control knob of the PPS to its full CCW position and turn the EM, VM, and PPS off.

7. With the PPS turned off, wire up circuit #2, using the 500 nF (0.500 μF) capacitor for C_3 and the 1000 nF (1.00 μF) capacitor for C_4. Use a mounted resistor for R_3 and the single-

pole-single-throw (SPST) knife switch for S_2. In order to minimize errors of procedure, place pieces of cardboard under C_3 and C_4 to identify each capacitor and to indicate the polarity of charge to be given to each, according to the circuit diagram.

8. Wire up circuit #3, using the central and end binding posts of one side of the DPDT switch for S_3. Use mounted resistors for R_4 and R_5. Clip the input leads of the EM to R_4 and R_5. Set the EM to the 10 V range. Use another pair of banana plug leads to connect the second PPS to the circuit.

9. Open S_2 and S_3. Turn the EM, VM, and PPS of circuit #2 on. Hold the red and black probes in contact and press the zero button to zero the EM. See note in step #2 if you have difficulty doing this.

10. Adjust the VOLTAGE control knob of the PPS of circuit #2 for an output of 5.0 V.

11. Close S_3. Turn on the PPS of circuit #3 and adjust its output for 3.0 V. With S_2 still open, use the probes to charge C_3 to that voltage, with the polarity shown on the cardboard. Maintain contact for about 5 seconds.

12. In a manner similar to step #10, charge C_4 to 4.0 V.

13. Close S_2 and open S_3. Use the probes to read V_3' and V_4', observing correct polarity. Record the VM readings in a table along with V_p. Open S_2.

14. Adjust the voltage of the PPS of circuit #2 for an output of 10.0 V.

15. Repeat steps 11-13.

16. Adjust the voltage of the PPS of circuit #2 for an output of 15.0 V.

17. Repeat steps 11-13.

18. Before disassembling the circuits, set the voltage and current output of both power supplies to zero and turn them off. Turn the EM and VM off. Carefully return equipment to the box and cart. All spade terminal to spade terminal wires should go into the wires' cart in the lab room. All other wires should be returned to the box and equipment cart.

III. CALCULATIONS

1. Make a table containing the quantities V_1, V_2 and V' for the four cycles of circuit #1. The values for V' should be 20 times the VM readings taken for circuit #1. V_2 is zero for the first cycle. For the remaining cycles, V_2 takes the value of V' from the preceding cycle. Leave room for the four values of C_1 to be calculated.

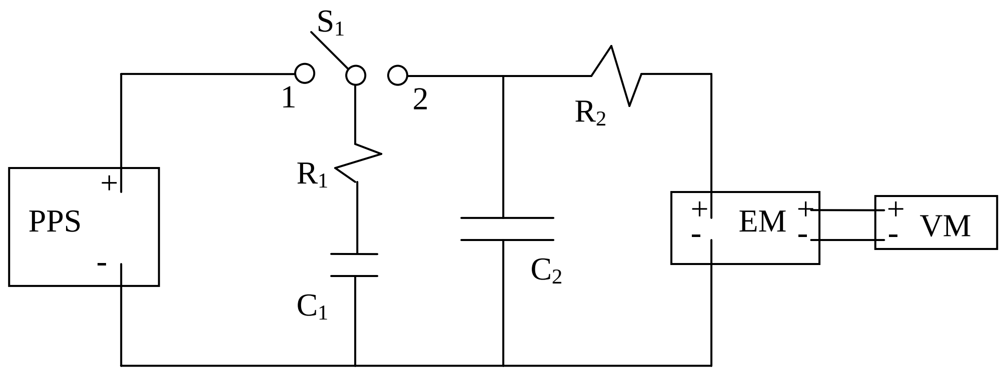
2. Calculate a value of C_1 for each set of voltages and add these values to the table of step #1. Use nF as units. Show the calculation once. Find the average experimental value of C_1. Compare this result with the nominal value, finding a percent difference.

3. Make a table containing the quantities V_p, V_3, V_4, V_3' and V_4' for the three trials of circuit #2. V_3' and V_4' should be 20 times the recorded VM readings.

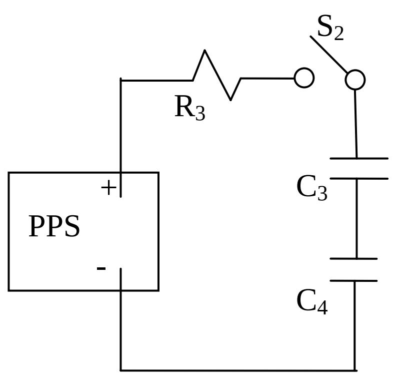
4. Calculate the theoretical values of V_3' and V_4' for the three trials. Show the calculation in one case. List in a table the experimental value, the theoretical value and the percent difference for V_3' and for V_4' for the three trials.

IV. DIAGRAMS

Circuit #1



Circuit #2



Circuit #3