

EXPERIMENT 3 MAGNETIC FORCE LAW

I. THEORY

To a good approximation, a uniformly magnetized rod behaves as if its magnetism were concentrated at two points near the ends of the rod, called poles. It is found by experiments that like poles of different magnets repel each other, while unlike poles attract. The two poles of a single magnet are always of equal strength; that is, they exert equal forces on a third pole located equally far from each of the two given poles.

The force of attraction or repulsion between poles is observed to decrease as the distance between the poles increases. Let us assume that the force varies inversely as some power of the distance r between the poles:

$$F = \frac{C_1}{r^n} \quad (1)$$

The purpose of this experiment is to determine the value of the number n in equation (1).

But is n actually constant? We can only answer this question experimentally. We will see that n depends in a simple way on the slope of a graph of the experimental data. If the slope is constant, that is, if the graph is linear, then n is a constant. If the slope is not constant, then n is not constant and equation (1) is probably not the best mathematical model of the force.

C_1 in equation (1) cannot be a universal constant, as it must depend on the strengths of the two poles. Its numerical value of C_1 is therefore of no interest to us.

As with electrostatic forces, the concept of a field is a useful one with magnetic forces. Pole #1 may be considered to create a magnetic field, which then acts on pole #2. Although modern textbooks define the magnetic field in terms of forces acting on moving electric charges, the older definition (which is still correct) is more useful in this experiment: magnetic field is the force acting on a test pole, divided by the strength of the test pole. (The similarity to the definition of the electric field vector E is obvious.) Applying this definition to equation (1), we conclude that the magnetic field created by a pole will have a magnitude which varies with the distance from that pole according to the equation

$$B = \frac{C_2}{r^n} \quad (2)$$

n in this equation must, of course be the same as in equation (1). C_2 clearly depends on the strength of the pole which creates the magnetic field, and is thus not a universal constant.

The magnetic field \vec{B}_N created by a North pole is directed away from the North pole, while the field \vec{B}_S created by a South pole is directed toward the South pole.

The resultant force exerted on a test pole by a bar magnet is the vector sum of the forces exerted by the two poles of the magnet. Therefore the resultant field created at any point is the vector sum of the two fields created by the poles of the magnet:

$$\vec{B}_m = \vec{B}_N + \vec{B}_S \quad (3)$$

The letters North and South refer to the north-seeking and south-seeking pole, respectively, while m refers to the magnet.

Let x be the distance between the center of a bar magnet and either pole. Consider a point P, located a distance r from the center of the bar magnet, along a straight line passing from the center of the magnet through the North pole. Let r be much greater than x. The distances from the North and South poles to P are r - x and r + x, respectively.

At the point P there are two fields: \vec{B}_N pointing away from the magnet, and \vec{B}_S pointing toward the magnet. Since these fields are antiparallel, the magnitude of the resultant field is given by

$$B_m = |B_N - B_S| \quad (4)$$

Using equation (2) twice, equation (4) becomes

$$B_m = \left| \frac{C_2}{(r-x)^n} - \frac{C_2}{(r+x)^n} \right| = C_2 \cdot \left| (r-x)^{-n} - (r+x)^{-n} \right| \quad (5)$$

We now expand the powers by using the binomial theorem. Using only the first two terms, we have

$$(r+x)^{-n} = r^{-n} - nxr^{-n-1} \text{ and } (r-x)^{-n} = r^{-n} - (-n)xr^{-n-1} \quad (6)$$

Taking the difference of these two and substituting into equation (5) yields

$$B_m = 2C_2nx/r^{n+1} = C_3/r^{n+1} \quad (7)$$

In order to determine n in equation (7), we must determine relative values of B_m for different values of r. This is done with the help of a magnetometer.

The magnetometer consists of a small magnetized body attached to a non-magnetic pointer. In some models the small magnetized body is a disk, while in other models it is a short compass needle. By convention, the non-magnetic pointer is mounted at right angles to the direction of magnetization of the disk or needle.

The magnetized body and pointer are balanced on a pivot and are therefore free to rotate in the horizontal plane. The magnetized body rotates until its North pole points in the

same direction as the horizontal component of the resultant magnetic field at that location. When the bar magnet is located far from the magnetometer, its field is negligible in comparison to the earth's field, and so the North pole of the magnetized body points toward magnetic north, and the non-magnetic pointer points toward the magnetic east and west. In this position the pointer reading is zero, when the magnetometer is properly aligned.

Suppose now that the bar magnet is placed with its center a distance r from the center of the magnetometer, in a direction toward the magnetic west. Suppose also that the N pole of the bar magnet points toward the magnetic east. Then there will be two horizontal fields at right angles at the center of the magnetometer: a field B_e due to the earth, pointing toward magnetic north, and a field B_m due to the bar magnet, pointing toward magnetic east. The N pole of the magnetized disk or needle will point in the direction of the resultant field, and the non-magnetized pointer will now indicate an angle θ instead of zero. A diagram shows that the magnitudes of the two fields and the angle are related by the equation

$$B_m = B_e \tan \theta \quad (8)$$

Combining equations (7) and (8) yields the equation

$$\cot \theta = (B_e / C_3) r^{n+1} = C_4 r^{n+1} \quad (9)$$

Taking logarithms, we obtain the equation

$$\log(\cot \theta) = \log C_4 + (n+1) \log r = (n+1) \log r + C_5 \quad (10)$$

$n+1$ is then the slope of a graph of $\log \cot \theta$ versus $\log r$. Therefore, the value of n may be found from the slope of a graph of $\log \cot \theta$ vs $\log r$. If such a graph is linear, then the slope and value of n are constant.

II. LABORATORY PROCEDURE

CAUTION: Do not tilt the magnetometer excessively, as the glass cover (on some models) may fall out and break. If you have a mechanical watch, keep it some distance from the bar magnet. Do not drop the magnet, as this would tend to demagnetize it.

1. Place the magnetometer on an inverted wooden box. Place the level on top of the glass cover of the magnetometer. Level the magnetometer.
2. Move the level and the bar magnet a considerable distance from the magnetometer. Remove any ferrous metal (for example, certain types of mechanical pencils) from the immediate vicinity of the magnetometer, as they will affect the readings. (Use the bar magnet to test any suspicious metals.) Turn the knob or wheel to raise the pivot until the pointer can move freely; however do not raise the pivot so high that the pointer is pushed against the glass cover. Rotate the magnetometer until both ends of the pointer read zero.

If this is not possible because of a bend in the pointer, adjust the magnetometer so that both ends of the pointer are off from zero by equal amounts. If the instrument has a mirror below the pointer, use it to eliminate parallax error, by placing your eye directly above the end of the pointer, so that the end of the pointer lines up with its image in the mirror. Again level the magnetometer.

3. If the magnetometer does not have a built-in meter stick, pass a thin meter stick through the brackets, and center it.
4. Place the bar magnet on the meter stick with its center 18 cm from the center of the magnetometer, and with its long axis parallel to the long axis of the meter stick. This is most easily accomplished by placing the magnet so that its two ends are equally distant from the 18 cm mark. Record the readings of both ends of the pointer, estimating to the nearest tenth of a degree.
5. Repeat step #4 with the center of the bar magnet located at distances of 21, 24, 27 and 30 cm from the center of the magnetometer.
6. Repeat steps 4 and 5 with the bar magnet rotated through 180 degrees. This will reverse the vector B_m and cause the pointer to deflect in the opposite sense; however, treat all angles as positive. Do not attempt to reverse the vector B_e .
7. Turn the knob or wheel to secure the pointer before returning the apparatus. If a meter stick was added to the magnetometer in step #3, remove it.

III. CALCULATIONS

1. Make a table with a column for each of the following quantities: r , $\log r$, θ , $\cot \theta$, and $\log \cot \theta$. The value of r should be in cm so that $\log r$ will take positive values. (Can you show that the graph of step #2 will have the same slope if r is changed from cm to m?) The value used for θ must be the average of all values recorded at each value of r .
2. Plot a graph of $\log \cot \theta$ versus $\log r$. The plotted points must cover a large part of both axes; this means that $(0, 0)$ should not be on the graph. Draw the straight line which best represents the plotted points.
3. Use the Method of Least Squares to calculate the slope of the graph. Include a calculator or computer printout, or list the four sums of page I-8 of this manual. Note the warning about rounding errors on page I-8.
4. Calculate the quantity n from the slope. Is the graph approximately linear? Is n constant within experimental error?
5. Round n off to the nearest integer. Express equations (1) and (2) as inverse proportions, using the proportionality symbol.