EXPERIMENT 15
PHYSICAL PENDULUM

I. THEORY

The purpose of this experiment is to measure the acceleration due to gravity by means of a physical pendulum.

The simple pendulum may be defined as a point mass attached to a massless unstretchable string, which is attached to a rigid support. Such a pendulum is, of course, only an idealization, but attaching a small dense sphere to the end of a long light string can make good approximation.

Any pendulum that is not a simple pendulum is, by definition, a physical pendulum or compound pendulum. A physical pendulum may be constructed by supporting a rigid body, symmetrical or otherwise, by some sort of pivot. The pivot may be located at any point except the center of mass.

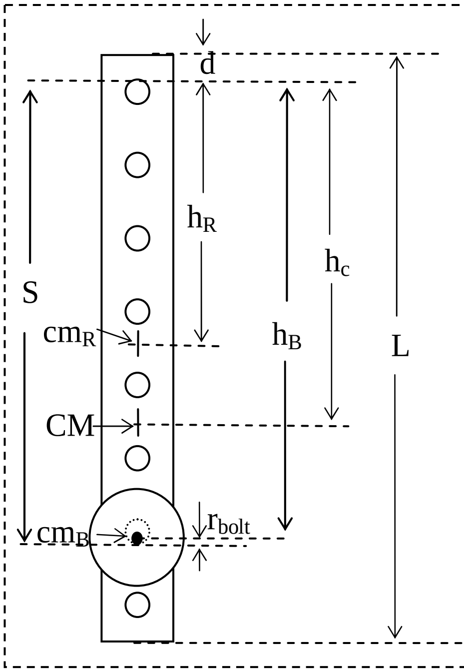
As in the case of the simple pendulum, the periodic motion is only approximately simple harmonic, because the restoring torque is proportional to the sine of the angle of displacement, rather than to the angle itself. For small amplitudes the motion may be considered Simple Harmonic Motion for all practical purposes. Under these conditions the theoretical period is given by

$$T = 2\pi\sqrt{\frac{I_p}{Mgh_c}}$$

In which I_P is the moment of inertia about the pivot point P, M is the mass of the pendulum, and h_c is the distance from the pivot point to the center of mass. This equation can be used to calculate an experimental value for the acceleration due to gravity using experimentally determined values for T, I_P, M, and h_c.

The apparatus for this experiment consists of a rod, a knife-edge pivot, and a cylindrical bob. A hole drilled through the rod, near one end, serves as the pivot point, while a number of other holes in the rod allow the bob to be attached to the rod at various distances from the pivot point.

Let the rod have mass M_R and length L. Let h_R be the distance from the pivot point to the center of mass of the rod. The figure on the following page shows the relationship between the distance h_R, the length of the rod, L, and the distance, d, from the end of the rod to the pivot point. We will assume that the center of mass of the rod lies at its center. The moment of inertia of the rod about the pivot point, I_R, may be calculated using the Parallel Axis Theorem. The finite width of the rod can be neglected, since it is quite small in comparison to the length. The effect of the holes in the rod on its moment of inertia can also be neglected.



$$h_R = \frac{L}{2} - d$$

$$h_B = S - r_{bolt}$$

$$h_c = \frac{M_R h_R + M_B h_B}{M_R + M_B}$$

Let the cylindrical bob have mass M_B and radius r_B. Let h_B be the distance from the pivot point to the center of the mass of the bob. The figure above shows the relationship between the distance h_B, the distance from the pivot point to the point of support of the bob, S, and the radius of the bolt holding the bob in place, r_{bolt}. Again, the moment of inertia of the bob about the pivot point, I_B, may be calculated using the Parallel Axis Theorem.

The total moment of inertia about the pivot point, $I_{P,}$ is then the sum of the moments of inertia of the rod and the bob about the pivot point, $I_P = I_R + I_B$.

The figure also shows the pivot point, the center of mass of the rod, cm_R, the center of mass of the bob, cm_B, and the combined center of mass, CM. In order to calculate the distance, h_c from the pivot point to the combined center of mass, we use the definition of the center of mass for two bodies.

II. LABORATORY PROCEDURE

1. Attach the knife-edge support to the table, using two clamps. To protect the wood surfaces from damage, place the <u>flat surface</u> of each clamp against the wood surfaces.

2. Use a two-meter stick and a sliding caliper jaw to measure the overall length L of the rod. Choose the hole at one end of the rod as the pivot hole. Use a vernier caliper to measure the distance d from the end of the rod to the pivot point.

3. Use a vernier caliper to measure the diameter of the cylindrical bob. Since the diameter of the bob varies somewhat, make several measurements and calculate an average value.

4. Disassemble the bob. Use the vernier caliper to measure the diameter of the bolt that holds the two sections together.

5. Locate the hole which is closest to 10 centimeters from the pivot hole. Using sliding caliper jaws on the meter-stick, measure the distance S (outside to outside) between this hole and the pivot hole. Attach the bob to the rod using this hole. Before tightening the bolt, hold the rod in the vertical position with the pivot hole of the rod upward. Tighten the bolt, after making sure it is resting on the bottom of the hole.

6. Place the pivot hole of the rod over the knife-edge. Set the pendulum into oscillation, with amplitude of about five degrees. Start the stopwatch as a pendulum comes to rest at one extreme position. Stop the stopwatch after 50 cycles. Record the elapsed time. Repeat for another 50 cycles.

7. Repeat steps 5 and 6 with the center of the bob at approximately 15, 20, 25, 35, 50, 70, and 100 centimeters from the pivot point. Measure and record the precise distance S in each case. The elapsed time will go through a relative minimum as the distance S increases.

8. Place the rod on the left pan of the large double pan balance. Determine its mass to the nearest gram by using standard masses and the sliding mass of the balance.

9. Disassemble the bob and determine its mass in two parts. Include the bolt with one part.

III. CALCULATIONS

1. Referring to the figure on the preceding page, calculate the values of h_B. For each value calculate the corresponding experimental period. List the values in a table.

2. Plot a graph of period versus distance between the pivot and the center of mass of the bob, h_B. You do not need to include the origin on your graph. Choose a scale so that the plotted points will cover a large part of the graph. Draw a smooth curve through the plotted points. By inspection, determine the values of h_B and period where the period is a relative minimum.

3. Using the graph, determine the period when the center of mass of the bob is 80.0 cm from the pivot. Through how many cycles would the pendulum oscillate in 5.00 min if the bob could be set at this position?

4. Calculate the total mass of the pendulum.

5. Calculate the distance h_R.

6. Calculate the moment of inertia of the rod about the pivot point.

7. Calculate the moment of inertia of the bob about the pivot point for the values of h_B which were approximately 25, 50, and 100 cm. Use the measured values of h_B in each case.

8. For the same three values of h_B, calculate the total moment of inertia of the pendulum about the pivot point.

9. For the same three values of h_B, calculate the distance h_C from the pivot point to the center of mass of the pendulum.

10. For the same three values of h_B, calculate the experimental value of the acceleration due to gravity. List the results of steps 7-10 in a table.

11. Calculate the average experimental value of g, and the percent error, using 980 cm/s^2 or 9.80 m/s^2 as the standard value.