

## EXPERIMENT 11 EQUILIBRIUM OF A RIGID BODY

### I. THEORY

A rigid body is said to be in equilibrium if it has neither linear acceleration nor rotational acceleration.

According to Newton's First Law, the absence of linear acceleration requires that the vector sum of all forces be zero

$$\sum \vec{F} = 0 \quad (1)$$

This is sometimes called the First Condition of Equilibrium.

According to Newton's Second Law for Rotation, the absence of rotational acceleration requires that the vector sum of all torques be zero.

$$\sum \vec{\tau} = 0 \quad (2)$$

This is sometimes called the Second Condition of Equilibrium.

It may be shown that if equation (1) is satisfied and equation (2) is satisfied for one axis, then equation (2) is satisfied for every axis.

The purpose of this experiment is to test the First and Second Conditions of Equilibrium.

In this experiment all forces lie in a single plane. One consequence of this is that the vector equation labeled (1) may be replaced by two scalar equations.

$$\sum F_x = 0 \text{ and } \sum F_y = 0. \quad (3)$$

Another consequence is that all torques are either clockwise or counterclockwise. The vector sum of equation (2) then becomes a scalar or algebraic sum, greatly simplifying the calculations.

In order to create a rigid body in equilibrium, we begin with a force table, which is a horizontal disk supported by a tripod base. We place a steel disk on the force table, thus forming a smooth flat surface. Then we place three ball bearings on the disk, and then a second steel disk on top of the ball bearings. In order to exert known forces on the upper disk, we insert pegs into holes drilled part way through the upper disk. Then we attach strings to the pegs, and run the strings over pulleys which are clamped to the force table. Finally, we hang weight hangers and standard masses from the strings.

## II. LABORATORY PROCEDURE

1. Remove a force table from storage and place it on the laboratory table. Remove the center pin, if one is present. Place the plain (undrilled) disk, unpainted side up, on top of the force table, being careful to avoid getting oil from the disk on your clothing. Place the other disk, with holes upward on top of the first disk. Place a level on top of the disk and level the force table. Remove the level and the top disk, and place three ball bearings on top of the undrilled disk, well separated from each other. Place the drilled disk on top of the ball bearings, with the holes upward. If the disk is well leveled, it should remain stationary when released. If the disk moves to one side after being released, adjust the level of the force table (you should not need the level).
2. Attach a spacer to each of the pulleys, in order to raise the pulleys when they are clamped to the force table. Clamp three of the pulleys at random positions on the force table. Place a sheet of paper (the data sheet) on top of the upper disk. Place three pegs in holes chosen at random in the upper disk, punching a hole in the data sheet with each peg. Run a string from each peg over a pulley and attach a weight hanger. If the strings touch the upper disk, add a 20 gm mass to each spacer, to raise the pulleys somewhat higher.
3. Increase the three masses so that all are different by at least 70 g and none are less than 250 g, including the 50 g weight hanger. Let the upper disk move until it is in equilibrium, moving a pulley, or changing the masses on the weight hangers if necessary. **THE UPPER DISK NEED NOT BE CENTERED OVER THE FORCE TABLE TO BE IN EQUILIBRIUM.** To minimize pulley friction, rotate each pulley about a vertical axis until each string goes straight over its pulley. After adjusting each pulley, you will need to check all of the other pulleys until all are aligned with their corresponding strings. You will not be reading angles from the force table in this lab, so the strings need not lie along radial lines.
4. Next to each string record the total mass hanging from that string, including the 50 g weight hanger. Make a pencil mark under each string at the point where that string goes off the data sheet. Remove the masses, the pegs, and the data sheet from the force table. Use a straightedge to draw in a line of action for each force, using the center of each hole and the pencil mark previously made as guides. Extend each line of action to both edges of the data sheet. The three lines of action should intersect at a single point, or be parallel, within experimental error. If they do not, have your instructor look at your data sheet before you continue. Draw an arrowhead on each line of action to indicate the direction of the force.
5. Take a new data sheet and repeat Steps 2-4, with four forces instead of three. Each of the four masses must again be different by at least 70 g, and none may be less than 250 g, including the weight hanger. Unlike the case of three forces, the four lines of action do not need to pass through a single point or be parallel.
6. Remove the spacers from the pulleys. Put the weight sets in order: 500, 2-200, 100, 50, 2-20 and 10 g.
7. Write all group members' names on both data sheets and give them to your instructor to photocopy.

### III. CALCULATIONS AND ANALYSIS

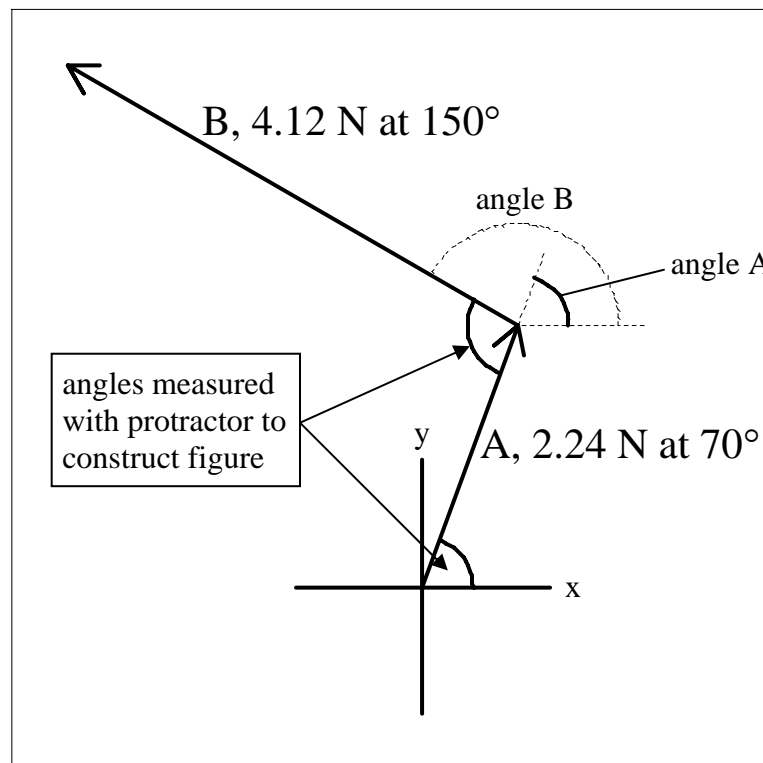
Use SI units for all forces and torques. The forces are the tensions in the strings, which are each equal to the weight of the mass attached to the string.

1. Test the First Condition of Equilibrium in the case of three forces by using component addition of the forces, as follows: Draw a line through the lines of action of the three forces on the data sheet. You can draw this line anywhere provided it crosses all three lines of action. This line will be the x-axis. Choose a positive end for the x-axis. Measure the angle between the positive x-axis and each force vector, measuring in the counter clockwise sense. Record the angles on the data sheet. Make a table (not on the data sheet) containing the columns Magnitude, Angle, X component, and Y component. Fill in the table with the values appropriate to the three forces. Calculate the algebraic sum of the x-components and the algebraic sum of the y-components. Is the first condition for equilibrium satisfied within experimental error?
2. Test the First Condition of Equilibrium in the case of four forces by constructing a force polygon on a separate sheet of unlined paper, as follows: First draw a set of x and y axes on this sheet.

Then, on the data sheet, draw a line passing through the lines of action of all four forces. This line will be the x-axis. Choose a positive end for the x-axis. Measure counter-clockwise, the angle each force makes with the positive x-axis. Record the angles on the data sheet. Label your forces A through D.

Next, on the unlined paper,

- Choose a scale, for example  $1 \text{ cm} = 10 \text{ N}$ , so that the polygon covers a substantial part of the sheet.
- Use a protractor to draw a line from the origin in the direction of vector A.
- Use a ruler to mark off a length corresponding to the magnitude of the vector, using your chosen scale.
- Place an arrowhead at the tip of this line.
- Use a protractor to draw a line from the tip of vector A in the direction of vector B, using the interior angle on the polygon (see the figure on the following page). You will have to calculate the interior angle.
- Repeat this process with the two remaining forces.
- Next to each vector record its name (A, B, C, or D), its magnitude in newtons, and its angle, as recorded on the data sheet.
- Measure the distance by which the polygon fails to close, and convert the distance into newtons. Apart from rounding and procedural errors, this force represents pulley and ball bearing friction.



3. In order to test the Second Condition of Equilibrium, choose a point at random on the data sheet with three forces. The point must be far from all of the lines of action or you will have large round off error. Try to choose a point 10 cm or more from every line of action; if you cannot locate a point at least 8 cm away from every line of action, consult with your instructor. Draw a perpendicular line from the point to each of the lines of action. The length of this line will be the lever arm for the corresponding force. Measure and label the length of each lever arm to the nearest millimeter. Make a table (not on the data sheet) containing the columns Force Magnitude, Lever Arm, Torque, and Sense (clockwise or counter clockwise). Fill in the values appropriate for all three forces. Calculate the total clockwise torque and the total counterclockwise torque. Find the percent difference between the absolute values of the two totals.
4. Repeat step 4 for a point chosen at random on the data sheet with four forces. Again, the point must be far from all of the lines of action.
5. Show why the lines of action for three forces must intersect in a single point or be parallel for the Second Condition of Equilibrium to be satisfied. Explain why this is not necessary in the case of four or more forces.