EXPERIMENT 3
THE ACCELERATION OF A ROLLING BODY

I. THEORY

The theoretical analysis of the motion of a body of circular cross-section rolling down an inclined plane is rather complex, and will not be undertaken until later in this course. The purpose of this experiment is to find purely experimental answers to certain questions which suggest themselves concerning such motion.

First, is the acceleration constant?

Second, does the acceleration depend on the mass of the body?

Third, does the acceleration depend on the diameter of the body?

Fourth, how does the acceleration depend on the angle of inclination of the plane?

Fifth, does the acceleration depend on the distribution of mass of the body, that is, whether the body is a sphere or a cylinder, solid or hollow?

We begin with the equation

$$x = \tfrac{1}{2} a t^2 \qquad \textbf{(1)}$$

which is the relation between position, acceleration and time for a body which accelerates at a constant rate, starting from rest at the origin at zero time.

From equation (1) we see that the distance traveled is proportional to the square of the elapsed time, under the conditions listed in the preceding paragraph. To test for constancy of acceleration, we will time a body as it rolls various distances from rest along an inclined plane. Then we will plot a graph of x versus t^2. If the graph is linear and passes through the origin, within experimental error, we may conclude that the acceleration is constant.

Assuming that the answer to the first question is affirmative, we may answer the remaining questions by determining the accelerations of rolling bodies of various masses, diameters and mass distributions for various angles of inclination of the plane. In all of these cases we will measure the elapsed time t for the body to roll a distance x. Then we will find the acceleration of the body by using the equation

$$a = \frac{2x}{t^2} \qquad \textbf{(2)}$$

which is another form of equation (1).

To answer the second question, we will roll two cylinders having equal diameters but different masses down the same inclined plane. Having found the two accelerations from equation (2), we will choose between the following three possibilities:

direct proportion:
$$\frac{a_1}{a_2} = \frac{m_1}{m_2} \quad \textbf{(3)}$$

inverse proportion:
$$\frac{a_1}{a_2} = \frac{m_2}{m_1} \quad \textbf{(4)}$$

independence:
$$\frac{a_1}{a_2} = 1.00 \quad \textbf{(5)}$$

Of course, other possibilities exist, but these three seem the most probable.

To answer the third question, we will roll two cylinders of different diameter down the same inclined plane. The three most probable relationships between acceleration and diameter are assumed to be direct proportion, inverse proportion, and independence. Thus we may use equations (3), (4) and (5), but with diameter D replacing mass m.

No one will doubt that the acceleration of a rolling body increases with the angle of inclination of the plane on which it rolls. In order to determine whether a direct proportion exists, we will measure the acceleration at several angles of inclination, and plot the acceleration against the three trigonometric functions which increase with increasing angle, namely the sine, the tangent and the secant. If variable y is directly proportional to variable x, then a graph of y versus x must be a straight line which passes through the origin.

To test whether the acceleration of a rolling body depends on the distribution of mass of the body, we will roll cylinders and spheres, both solid and hollow, down the same inclined plane. (A quantitative method of expressing the distribution of mass of a body will be developed later in this course.)

II. LABORATORY PROCEDURE

1. Take a plank out of storage, and measure and record its length from end to end (not from "0" to "225 cm").

2. Place the "zero" end of the plank toward the sink. Use three rods, two right-angle clamps and two tripod bases to construct "goal posts" to support the "zero" end of the plank at a height h of 20.0 cm above the laboratory table. Measure the height h as in the diagram, so that the angle of inclination can later be calculated; do <u>not</u> measure to the "zero" mark

on the plank. For fine adjustment of the height, slide the plank relative to the supporting rod, rather than raising or lowering the rod.

3. Use the following list to determine which rolling body to use in each of the remaining steps of the experiment:

A	large diameter short solid cylinder
B	large diameter long solid cylinder
C	small diameter solid cylinder
D	hollow cylinder
E	solid sphere
F	hollow sphere

4. Make a table with the following columns on your data sheet: Body, Distance x, Elevation h, t_1, t_2, t_3.

5. Place body A at the zero mark on the plank. Release the body and simultaneously start the stopwatch. Stop the stopwatch at the instant the body crosses the 20 cm line. These three operations must be carried out by the same person in order to minimize errors due to reaction time. Record the time to the nearest 0.01 sec. Carry out two additional runs in the same way.

6. Using the same rolling body, repeat step 5 for all of the remaining distances on the plank, always starting at the zero mark.

7. Switch to body B and carry out three runs as before, but using only the 225 cm distance.

8. Repeat step 7 using bodies C, D, E and F. If either of the spheres continually rolls off the same side of the plank, raise that side of the plank by rotating the support structure about a vertical axis. If a sphere rolls off either side of the plank with equal frequency, try turning the plank over; some planks have lines marked on both sides. If none of the above methods solves the problem, use a section of drain pipe as a sphere guide, but hold it firmly so it doesn't rock, as that would absorb energy from the sphere, slowing it down.

9. Elevate the "zero" end of the plank 25.0 cm above the laboratory table. Using body A, repeat step 7.

10. Repeat step 9 for plank elevations of 30.0, 35.0 and 40.0 cm.

11. Use a vernier caliper to measure the diameters of bodies A, B and C, estimating to the nearest 0.01 cm.

12. Use a triple beam balance to measure the masses of bodies A and B, estimating to the nearest 0.1 g.

III. CALCULATIONS AND ANALYSIS

1. Average the three values of t for each x. Make a table of the quantities x, t and t_2 using the data of II-5 and II-6 only. Include x = 0 at t = 0.

2. Plot a graph of x versus t_2. Draw the straight line which passes through the origin and best fits the plotted points. Does x appear to be proportional to t_2, allowing for experimental error? Does a body rolling down an inclined plane appear to accelerate at a constant rate?

3. Make a table with the following columns: Body, Elevation, Average time, Acceleration. In the table list the ten cases in which x = 225 cm. Show the calculation of the acceleration in one case.

4. In order to determine whether the acceleration of a body depends on the mass of the body, compare bodies A and B in the table of step 3, using only the elevation of 20 cm. Make a table containing the following quantities: a_1, a_2, a_1/a_2, m_1, m_2, m_1/m_2, and m_2/m_1. Express the ratios in decimal form, to a precision of three significant figures. Which of the three equations, (3), (4) and (5), is closest to being satisfied? Does the acceleration of a rolling body appear to be directly proportional to mass, inversely proportional to mass, or independent of mass?

5. In order to determine whether the acceleration of a rolling body depends on the diameter of the body, compare bodies A and C of the table of step 3, using only the elevation of 20 cm. Make a table containing the following quantities: a_1, a_2, a_1/a_2, D_1, D_2, D_1/D_2, and D_2/D_1. Does the acceleration of a rolling body appear to be directly proportional to diameter, inversely proportional to diameter, or independent of diameter?

6. In order to determine whether acceleration is directly proportional to some trigonometric function of the angle of inclination of the plank, make a table with the following columns: h, a, θ, $\sin(\theta)$, $\tan(\theta)$, and $\sec(\theta)$. List in the table the appropriate quantities for body A for each of the elevations from 20 to 40 cm. Show the calculation of θ in one case.

7. Plot a graph of a versus $\sin(\theta)$. Plot another graph of a versus $\tan(\theta)$. These graphs may be plotted on the same axes if different symbols or colors are used to distinguish them.

8. Plot a graph of a versus $\sec(\theta)$. Let the X axis extend from 0 to beyond 1, so that it can be determined whether the graph passes through the origin.

9. Within experimental error, can one or more of the three graphs of steps 7 and 8 be described as straight lines which pass through the origin? If so, which one? Can we conclude that the acceleration is directly proportional to the trigonometric function plotted, for the small angles used in this experiment.

10. Considering the fact that two of the three trigonometric functions plotted approach infinity for angles approaching ninety degrees, which function could acceleration reasonably be proportional to for both large and small angles?

11. From the table of step 3:

    a. Which is faster, a solid sphere or a hollow sphere?

    Use the average of the accelerations of the three solid cylinders to answer the next two questions.

    b. Which is faster, a solid cylinder or a hollow cylinder?
    c. Which is faster, a solid sphere or a solid cylinder?