Figure 1 shows a bottom view of the spherometer.

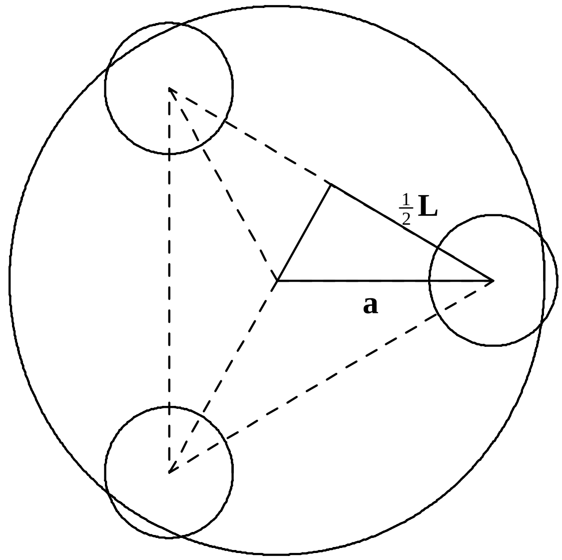

Figure 1

Let **L** equal the center-to-center separation of the feet. **L** is the difference between the outer-to-outer separation of the feet and the diameter of the feet, both of which are measured.

Let **a** be the distance from the center of each foot to the center of the system **P**.

In the right triangle shown in the figure, $\frac{1}{2}\mathbf{L} = \mathbf{a}\cos(30^\circ) = \frac{\sqrt{3}}{2}\mathbf{a}$. So, $\mathbf{a} = \frac{1}{\sqrt{3}}\mathbf{L}$.

Figure 2 shows one foot of the spherometer in contact with a sphere of unknown radius.

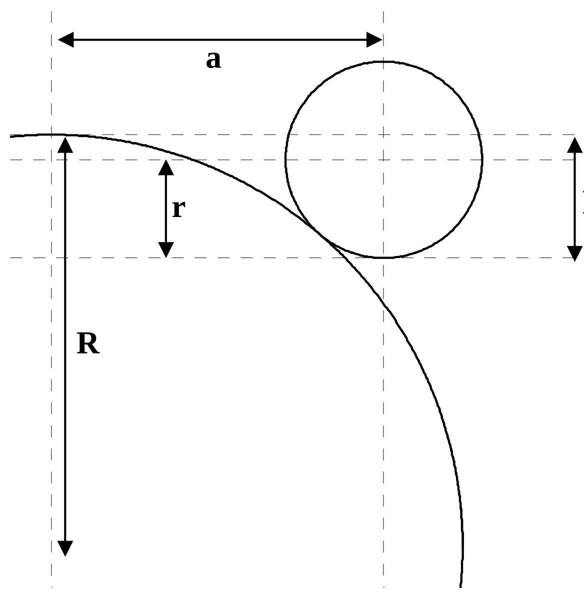

Figure 2

In this figure, **R** is the unknown radius of the sphere, **r** is the radius of the spherometer foot, and **h** is the difference between the spherometer reading when in contact with the sphere and the reading when in contact with a flat surface.

Figure 3 shows a right triangle superimposed on Figure 2.

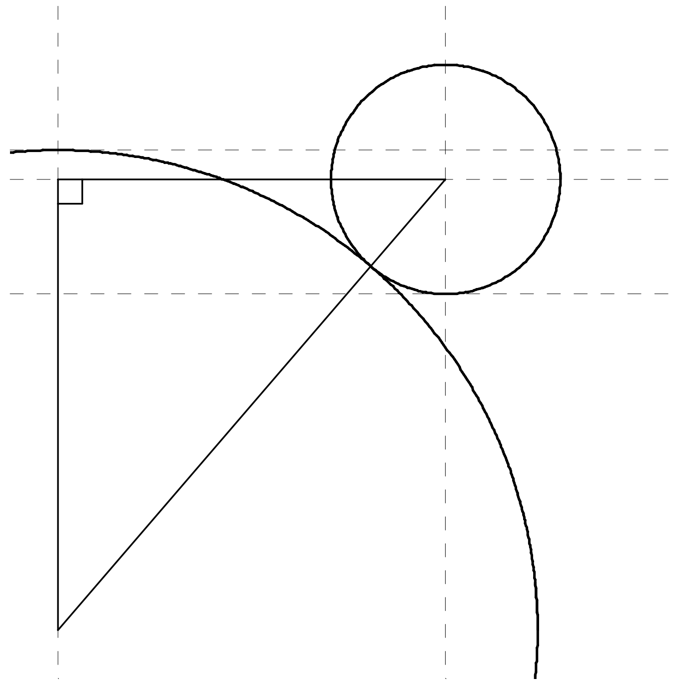

Figure 3

With the help of Figure 2, it is found that the length of the hypotenuse of the right triangle is $\mathbf{R}+\mathbf{r}$. The lengths of the two legs are $\mathbf{R}-\mathbf{h}+\mathbf{r}$ and **a**.

Applying the Pythagorean Theorem, $(\mathbf{R}+\mathbf{r})^2 = ((\mathbf{R}+\mathbf{r})-\mathbf{h})^2 + \mathbf{a}^2$.

$$(\mathbf{R}+\mathbf{r})^2 = (\mathbf{R}+\mathbf{r})^2 - 2\mathbf{h}(\mathbf{R}+\mathbf{r}) + \mathbf{h}^2 + \mathbf{a}^2$$

$$0 = -2\mathbf{h}(\mathbf{R}+\mathbf{r}) + \mathbf{h}^2 + \mathbf{a}^2$$

$$2\mathbf{h}(\mathbf{R}+\mathbf{r}) = \mathbf{h}^2 + \mathbf{a}^2$$

$$\mathbf{R}+\mathbf{r} = \frac{\mathbf{h}}{2} + \frac{\mathbf{a}^2}{2\mathbf{h}}$$

$$\mathbf{R} = \frac{\mathbf{h}}{2} + \frac{\mathbf{a}^2}{2\mathbf{h}} - \mathbf{r}$$

Replacing $\mathbf{a}^2$ with $\frac{1}{3}\mathbf{L}^2$ gives our final result: $\mathbf{R} = \dfrac{\mathbf{h}}{2} + \dfrac{\mathbf{L}^2}{6\mathbf{h}} - \mathbf{r}$.