

EXPERIMENT 15 THIN LENSES AND TELESCOPES

I. THEORY

It is remarkable that a single equation, the Gaussian Lens Equation, is capable of describing the properties of converging and diverging lenses, as well as convex and concave mirrors, including both real and virtual objects, and real and virtual images. This equation is usually written

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

In this equation, the focal length f is positive for converging lenses and concave mirrors and negative for a diverging lenses and convex mirrors. The object distance, s , is positive for real objects and negative for a virtual objects. The image distance, s' , is positive for a real images and negative for a virtual images.

The way in which the focal length of a thin lens depends on its refractive index and on its two radii of curvature, R_1 and R_2 , may be deduced by applying Snell's Law to the two refractions. In order to obtain a simple result, we must also assume that all angles of incidence are small. When this is done, we arrive at the Lensmaker's Equation

$$\frac{1}{f} = \left(\frac{n_b}{n_a} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

The indexes of refraction, n_a and n_b , are for the medium outside the lens and of the lens respectively. In many situations, n_a is the index of refraction of air which is approximately unity. The sign for R_1 or R_2 is positive if the center of curvature is on the same side as the refracted light and negative if on the opposite side.

The radius of curvature of the surface of a lens may be measured with the help of a spherometer. A spherometer consists of three identical spherical feet and a micrometer screw, held together by a frame. Each foot has a radius r and is located a distance L from each of the other feet (measured center to center). The micrometer screw is equidistant from the three feet, and moves in a direction perpendicular to the plane formed by the feet. To use the spherometer, we back off the micrometer screw, hold the feet firmly against the surface, and then rotate the micrometer screw until its tip makes firm contact with the surface. This is done twice, first with a plane surface, and then with the curved surface under study. If H is the **absolute value** of the **algebraic** difference between the two readings, then the radius of curvature of the curved surface is given by the equation

$$R = \frac{L^2}{6H} + \frac{H}{2} - r$$

A derivation of this equation can be found in the appendix of this lab manual.

A very useful principle in the study of optical systems is that the image of one mirror, lens, or surface serves as the object of the next mirror, lens, or surface. The image may

be real or virtual, as may the object. Thus, a complex optical system may be analyzed by repeated applications of the Gaussian Lens Equation.

The refracting astronomical telescope consists of a long focus positive objective lens, of focal length f_1 , and a short focus positive eyepiece, of focal length f_2 . For an object at infinity, the lenses should be separated by a distance $L = f_1 + f_2$ for minimum eyestrain. The eyepiece then forms a virtual image at infinity of the real image formed by the objective lens.

The Galilean Telescope, or opera glass, consists of a long focus positive objective lens (f_1) and a short focus negative eyepiece (f_2). For an object at infinity, and for minimum eyestrain, the distance between lenses is also given by $L = f_1 + f_2$. Since f_2 is a negative number for this telescope, the distance between lenses is less than f_1 , meaning that the eyepiece intercepts the converging rays of light before they can form a real image. The real image produced by the objective lens exists only in a mathematical sense, but nevertheless serves as a virtual object for the eyepiece, which forms a virtual image at infinity.

The angular magnification M of a telescope is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye by the original object. The theoretical magnification of both types of telescopes described above, for object and image at infinity, is given by $M = -f_1/f_2$. The negative sign is needed in this equation to conform to the convention that angular magnification is positive for an erect final image, and negative for an inverted final image.

In this experiment, we will test the Gaussian Lens Equation, measure the focal lengths of positive and negative lenses, measure the index of refraction of a symmetric converging lens, and study the angular magnification of telescopes.

II. LABORATORY PROCEDURE

NOTE: For large images on the screen, you may find that the inner part of the image does not focus at the same place as the outer part of the image. This is a result of spherical aberration. In this case, focus the center of the image on the screen.

1. Make a table for your data containing columns for Step Number (2, 3 ... 10, 11), object location, image location, first lens location, and second lens location. The last column will be blank except for in Steps 6 and 7, since these are the only two steps that will use two lenses.
2. Mount the light source to the optics bench at the zero position. Place the long focus converging lens (labeled D) at the 65.0 cm mark. Move only the screen to focus the image. You may use the short focusing lens as a magnifying glass to make sure the image is in sharp focus. Record the position of the object, the image (screen) and the lens.
3. Leaving the screen in position move the lens toward the light source until a sharply focused image appears again. Record the position of the object, the image and the lens.

4. Repeat step 2 with the lens at 60.0 cm.
5. Repeat step 2 with the lens at 55.0 cm.
6. Leave the long focus converging lens in place and place the screen at the 105.0 cm mark. Place the diverging lens between the converging lens and the screen. Move only the diverging lens until the image comes into sharp focus on the screen. Record the position of the object, the image and both lenses.
7. Repeat step 6 with the screen at 115 cm. Remove both lenses.
8. Place the short focus converging lens (labeled B) at the 12.0 cm mark. Move only the screen to focus the image. Record the position of the object, the image and the lens.
9. Leaving the screen in position move the lens toward the screen until a sharply focused image appears again. Record the position of the object, the image and the lens.
10. Repeat step 8 with the lens at 13.0 cm.
11. Repeat step 8 with the lens at 15.0 cm.
12. Set up the refracting astronomical telescope as follows: Remove all components from the optical bench except the two positive lenses. Place the short focus lens at zero, and the other positive lens at 25 cm. Place your eye just behind the short focus lens and look through the telescope at the scale projected on the screen. Move the long focus lens to focus on the scale. With one eye look through the telescope at the scale; with the other eye look directly at the scale. Estimate how many times larger the scale appears when seen through the telescope than when seen with the unaided eye. This is the angular magnification. If the image is upright, record the magnification as positive. If the image is inverted, record the angular magnification as negative.
13. Set up the Galilean Telescope by replacing the eyepiece in the refracting telescope with the diverging lens, using the same objective lens. Place the objective lens at about 10 cm on the optical bench. As before, move the objective lens to focus on the scale, and estimate the angular magnification. If the image is upright, record the magnification as positive. If the image is inverted, record the angular magnification as negative.
14. Use the vernier caliper to measure the diameter of one of the feet of the spherometer. Also, measure the distance between each pair of feet, outside to outside (three measurements). Average these three measurements and subtract the foot diameter to obtain the distance L , the center to center separation of the feet.
15. Back off the micrometer screw of the spherometer about a revolution. Hold the feet firmly in contact with the plane glass plate. Slowly rotate the screw until the tip makes solid contact with the plate. Take the reading, noting that the major scale is in mm and the minor scale is in hundredths of mm. If the screw went past zero, read the red scale and consider the reading as negative. If the colors are faded, the red scale is the *lower* half of the major scale, and the *inner* scale of the two minor scales. Repeat this procedure several times, until consistent readings are obtained, and keep only the best value.

16. Back the spherometer off about 1.5 revolutions and hold it in contact with one side of the short focus positive lens. Take the reading, which should be positive. Repeat, using the other side of the same lens. Record both readings.
17. Calculate H , the absolute value of the algebraic difference between the results of steps 15 and 16. It should be somewhat less than one mm.

III. CALCULATIONS AND ANALYSIS

1. For the long focus converging lens make a table of object distance, image distance and focal length. Use all of the data from Steps 2-5 of the procedure. In your table, how are the object distance and image distance of Step 2 related to the object distance and image distance of Step 3? Explain.
2. Repeat Calculation 1 for the short focus converging lens, using all of the data from Steps 8-11 of the procedure.
3. Using the calculated values for focal length in your tables, determine the average focal length of each of your converging lenses.
4. Make a diagram showing the data of Step 5 of the procedure, including the long focus converging lens L_1 , the object P_1 , and the image P_1' . Include the numerical values of the positions of all components. Directly below this diagram make another diagram showing all of the components of Step 6 in the procedure. Include both lenses (L_1 and L_2), both objects (P_1 and P_2), and both images (P_1' and P_2'). The real image of the first lens is the virtual object of the second lens, and should be shown in the same horizontal position in both diagrams. Include the numerical values of the positions of all components.
5. Using the diagrams above, calculate the object distance s_2 and the image distance s_2' for the diverging lens. Use these values to calculate the focal length of the diverging lens.
6. Repeat Calculation 5 using the data from Steps 5 and 7 of the procedure. No diagram is necessary.
7. Make a table listing the object distance s_2 and the image distance s_2' for the diverging lens and the focal length of the diverging lens. Average the two focal lengths.
8. Calculate the theoretical angular magnification of the refracting astronomical telescope, using the calculated focal lengths of the lenses. Find the percent difference between this value and the experimental value.
9. Repeat Calculation 8 for the Galilean Telescope.
10. Calculate the radii of curvature of the two sides of the short focus converging lens, using the data from the spherometer measurements. Make a sketch of the lens. In your sketch, label the two surfaces of the lens 1 and 2. Place dots, C_1 and C_2 , at the center of curvature for each side. Using this and the calculated focal length, determine the index of refraction of the short focus converging lens. Was this lens made of crown glass (1.52) or flint glass (1.66)?