

## EXPERIMENT 13 STANDING WAVES ON A STRING

### I. THEORY

The purposes of this experiment are to produce standing waves on a vibrating string, study the relation between string tension and wavelength of such waves and calculate indirectly the frequency of vibration of the string.

The ratio of mass to length of a string is called linear density and is represented by the Greek letter mu,  $\mu$ . Linear density is thus defined by the equation

$$\mu = \frac{m_s}{L_s} \quad (1)$$

in which  $m_s$  and  $L_s$  could be measured for *any* length of the string. To avoid cutting the string, we will use the entire length, a little less than two meters.

If a string is under tension, connected to two supports, and a disturbance is produced near one end, as by striking the string, a wave pulse will travel to the other end and be reflected with reduced amplitude back to the first end. For an idealized perfectly flexible string (one with no stiffness), Newton's Second Law predicts that the velocity of such a wave is given by the equation

$$v = \sqrt{\frac{F}{\mu}} \quad (2)$$

in which  $F$  is the tension in the string. This equation yields approximately correct results for real strings which are not too thick.

The tension will be varied in this experiment by passing one end of the string over a pulley and hanging a standard mass  $M$  from the end. The tension is then given by the equation

$$F = Mg \quad (3)$$

In order to set the string into vibration we attach the other end of the string to a flat steel rod. Alternating electric current in a coil adjacent to the rod magnetizes and attracts the rod twice during each cycle of AC current. Between attractions the rod springs back to its original position. Since there are 60 cycles of AC current in each second, the rod will vibrate with a frequency of 120 cycles per second, called 120 hertz (Hz).

The vibrating rod causes a series of wave pulses, called a traveling wave, to travel to the other end of the string with a velocity given by equation (2). Each of these pulses

reflects back toward the rod. When the original traveling wave combines with the reflected traveling wave, a standing wave is produced.

Standing waves are characterized by a series of points called *nodes*, where the amplitude is ideally zero, alternating with points called *antinodes*, where the amplitude is a maximum.

A *loop* is a section of the vibrating string between any two adjacent nodes. The *wavelength*, represented by the Greek letter lambda,  $\lambda$ , is equal to the total length of two loops, and is therefore the distance from one node to the second node over. In order to measure wavelengths as accurately as possible, we will measure the total length,  $L_n$ , of  $n$  loops, where  $n$  is as large as possible. The quantity  $L_n/n$  will be equal to one-half of the wavelength, so the wavelength is given by the equation

$$\lambda = \frac{2L_n}{n} \quad (4)$$

Wave velocity, wavelength and frequency for any kind of wave are related to each other by the equation

$$v = f\lambda \quad (5)$$

From equation (5) we see (after squaring the equation) that the square of the wavelength is directly proportional to the square of the wave velocity, since the frequency is constant in this experiment. From equation (2) we see that the square of the wave velocity is, in turn, directly proportional to the tension, since the linear density  $\mu$  is constant in this experiment. It follows that the square of the wavelength should be directly proportional to the tension. We will test this prediction by plotting a graph of wavelength squared versus tension. Such a graph should yield a straight line passing through the origin.

## II. LABORATORY PROCEDURE

1. It will be necessary for this lab to roll one of the center tables in the room to the end of one of the peninsula lab stations and create an 8-foot table edge. Unlock the wheels of one of the center tables in the room. Then, roll it so that its shorter end is against a peninsula lab station and one of its corners is touching one of the corners of the peninsula lab station.
2. Attach a table clamp near one end of the laboratory table. Attach a second table clamp approximately 5 feet from the first. One clamp should be on the movable table and one on the stationary peninsula station. Attach a vertical rod to each table clamp, and a right angle clamp to each rod. Attach a pulley to the support at the end of the table. Attach the vibrator to the other support.
3. Tie one end of the string to the hole at the end of the vibrating rod. Use a knot that can be untied easily so that it will not be necessary to cut the string later.

4. Run the other end of the string over the pulley and hang a 50 gram weight hanger from the end. Place a 50 gram mass on the weight hanger and record the mass  $M$  as 100 grams.
5. Plug in the vibrator. Adjust the effective length of the string by moving the vibrator and/or pulley until sharp nodes appear and the amplitude of the standing wave is maximum. Use as much of the string as possible, consistent with obtaining a maximum amplitude.
6. Place two sliding caliper jaws on a two-meter stick, one near each end. The jaw near the zero end of the stick should be placed at an even number, such as 10 cm, and locked in position. With one person holding each end of the stick, measure and record the total length,  $L_n$ , of as many loops as possible. The pulley end of the string may be taken as a node, but the end attached to the vibrating rod is neither a node nor an antinode. Be sure to subtract the reading of the jaw near the zero end of the stick from the reading of the other jaw to obtain the length. Also be sure to record the number,  $n$ , of loops spanned.
7. Repeat steps 3-5 using a mass of at least 200 gm.
8. Repeat step 6 using a mass of least 300 gm. This mass should exceed the mass of step 6 by at least 100 gm.
9. Untie the string from the rod of the vibrator and remove all knots. If you are unable to remove a knot, ask your instructor for assistance. Stretch the string out under moderate tension along a two-meter stick. Record the overall length  $L_s$ , estimating to the nearest cm.
10. Measure the mass of your string to the nearest 0.001 g using a balance.

### III. CALCULATIONS AND ANALYSIS

1. Calculate the linear density of the string in kg/m.
2. Make a table containing the columns Hanging Mass, Tension, Wave Velocity, Number of Loops, Length of  $n$  Loops, Wavelength, and Calculated Frequency.
3. Average the three values of the calculated frequency and find the per cent error with the expected value of 120 Hz.
4. Make a table containing the quantities Tension, Wavelength, and Wavelength Squared.
5. Plot a graph with Wavelength Squared on the vertical axis and Tension on the horizontal axis. Using a straightedge, draw the straight line which best represents the plotted points and passes through the origin. Is the graph linear, within experimental error? Does the square of the wavelength appear to be directly proportional to the tension, within experimental error, as predicted by equations (2) and (5)?
6. Calculate the slope of your graph and from it calculate the frequency of vibration.