EXPERIMENT 12
LINEAR SIMPLE HARMONIC MOTION

I. THEORY

The purposes of this experiment are to measure the force constant of a spring, to verify the theoretical equation for the period of a mass oscillating in simple harmonic motion on the spring, and to determine the effective mass of the spring under such conditions.

The theoretical period of a system composed of a mass M oscillating at the end of a massless spring of force constant k is given by

$$T = 2\pi\sqrt{\frac{M}{k}}\ .$$

Since no spring is massless, it would be more correct to use the equation

$$T = 2\pi\sqrt{\frac{M + m_s}{k}}\ ,$$

in which m_S is the effective mass of the spring, which is substantially less than the actual mass of the spring (M_S). Each coil of the spring oscillates with a different amplitude. The coil at the top of the spring has zero amplitude. The coil at the bottom of the spring has the same amplitude as the attached mass (M). The effective mass of the spring would be equal to its actual mass, if each coil had the same amplitude as the attached mass. The effective mass of a spring which is uniform along its length (not tapered or distorted by use) is equal to one-third of its actual mass. For this experiment, we will assume this relation is true for our springs.

The force constant of the spring is defined by Hooke's Law:

$$\vec{F} = -k\,\vec{x}\ .$$

Using this law, we will also determine a value for the force constant.

II. LABORATORY PROCEDURE

1. Suspend the spring (small end up for all parts of the lab) using the bar attached to the table clamp, the right-angle clamp and the threaded bar. Placing the right angle clamp at the top of bar will keep the weight hanger from hitting the floor in step 9. Hanging the spring at the threaded end will keep it from sliding during the experiment. Attach a weight hanger to the spring.

2. Using a 1-meter or 2-meter stick and a caliper jaw measure the height of the bottom of the weight hanger above the laboratory floor. Record the reading and a mass of 50 g in a table.

3. Add 50 g to the weight hanger. Measure the height of the bottom of the weight hanger above the floor. Record the height and the mass in your table.

4. Repeat step 3 until a total of 350 g has been <u>added</u> to the weight hanger.

5. Place 50 g on the 50 g weight hanger. Make a table containing three columns: attached mass, time for 50 oscillations-first run, and second run. Record the attached mass as 100 g. Raise the mass a few centimeters above equilibrium, and release it. If the amplitude is too large, the motion tends to be unstable. If the mass is not oscillating up and down along a straight line, start the mass again. Start the stopwatch as the mass reaches either extreme position. Stop the stopwatch after 50 complete oscillations. (One complete oscillation brings the mass back to the extreme position from which is started.) Record the elapsed time in your table. Carry out the second run in the same way, having a different student do the timing. If the times for your two runs differ by more than half a second, carry out two additional runs. If these runs differ by more than half a second, inform your instructor.

6. Repeat step 5 for total masses of 200, 300, and 400 grams.

7. With a balance, measure the actual mass of the spring.

8. OPTIONAL: If time permits, repeat steps 1-4 using <u>two springs</u> in <u>parallel</u>. To have the springs in parallel, hang both from the horizontal rod. To the bottom of each, attach the wooden piece, and to the wooden piece attach the weight hanger.

9. OPTIONAL: If time permits, repeat steps 1-4 using <u>two springs</u> in <u>series</u>, but stop once the mass added to the weight hanger reaches 250g. To have the springs in series, hang one from the horizontal rod. To the bottom of the first, attach the other spring, and to it the weight hanger.

10. Before returning your equipment, put the weight sets in order: 500, 2-200, 100, 50, 2-20, and 10 g.

III. CALCULATIONS AND ANALYSIS

1. Make a table containing mass added to the weight hanger and stretch of the spring. The stretch of the spring is how far the spring has stretched from its length when the weight hanger is empty.

2. Make a graph of stretch of the spring versus mass added to the weight hanger. Draw a best fit line. Is Hooke's Law verified? Calculate the slope of your line. From your slope, calculate the force constant of the spring.

3. Make a table containing the quantities: attached mass, experimental period, theoretical period, and percent difference. Record the four values of attached mass used in the step 5 and 6 of the procedure. Calculate and list in a table the experimental period for each mass; use the average time of your two runs. Using the force constant found in the previous step, calculate the theoretical period for each mass; **do not assume the spring was massless**. Determine the percent difference between each theoretical period you have calculated and the experimental period.

4. If you performed the optional procedural step 8, repeat calculations 1 and 2 for the data with two springs in parallel to find the equivalent force constant of the two springs. How does the equivalent force constant relate to the force constant of the single spring?

5. If you performed the optional procedural step 9, repeat step 4 for the springs in series.