Kinematics Problem Solving II

Frequently kinematics problems will be solved algebraically using the definitions and equations of motion for constant acceleration.

Definitions: $v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ $a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

Equations of motion for constant acceleration:

$$(0)\quad x_f = x_i + \tfrac{1}{2}(v_i + v_f)t \qquad (2)\quad v_f = v_i + a\,t$$
$$(1)\quad x_f = x_i + v_i\,t + \tfrac{1}{2}a\,t^2 \qquad (3)\quad v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Or

$$(0)\quad x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad (2)\quad v = v_0 + a\,t$$
$$(1)\quad x = x_0 + v_0\,t + \tfrac{1}{2}a\,t^2 \qquad (3)\quad v^2 = v_0^2 + 2a(x - x_0)$$

Kinematics problems may also be analyzed by using freeze-frame diagrams (pictures of the objects at different states) or graphical techniques.

The first problem will be solved as an in-class exercise using graphical and algebraic methods:

1. A student is running to catch the bus, which is stopped at the bus stop. She is running at a constant speed of 5.0 m/s. When she is still 80 m from the bus, it begins to pull away. The bus moves with a constant acceleration of 0.10 m/s².

   There are two approaches we will take to set up solution for this problem, depending on how the question to be answered is phrased.

   The first way the question may be asked is:
   > How far must the student run to catch the bus? How much time does this take?

   The second way the question may be asked is:
   > How far from the bus stop is the bus when the student catches up to it? How much time does this take?

   We will first look at a graphical solution to the problem(s) and then solve algebraically.

2. An antelope moving with constant acceleration covers the distance between two points 80 meters apart in 6.0 s. Its speed as it passes the second point is 15 m/s.
   (a) What is its speed at the first point?
   (b) What is the acceleration?

3. A sled starts from rest at the top of a hill and slides down with a constant acceleration. At some later time it is 32 meters from the top; 2.0 s after that it is 50 m from the top; 2.00 s later it is 72 m from the top; and 2.00 s later it is 98 m from the top.
   (a) What is the magnitude of the average velocity of the sled during each of the 2.0-s intervals after passing the 32-m point?
   (b) What is the acceleration of the sled? (The average velocity of sled for each interval of time is the same as the instantaneous velocity at the middle of that time interval.)
   (c) What is the speed of the sled when it passes the 32-m point?
   (d) How much time did it take to go from the top to the 32-m point?
   (e) How far did the sled go during the first second after passing the 32-m point?

4. At the instant a traffic light turns green, a car that has been waiting at the intersection starts ahead with a constant acceleration of 2.00 m/s^2. At the same instant a truck, traveling with a constant speed of 14 m/s, overtakes and passes the car.
   (a) How far beyond the starting point does the car overtake the truck?
   (b) How fast is the car traveling when it overtakes the truck?

5. The winner of the 100-m dash had a time of 10.0 s. The second-place runner had a time of 10.5 s. Assume the runners traveled at their average speeds of the entire 100 meters. How far behind the winner was the second-place runner when the winner crossed the finish line?

6. Repeat Problem 5, but this time assume that the runners started from rest and that each has a constant acceleration.

7. A skateboarder is moving at a constant speed of 3.5 m/s when she starts up a ramp. She undergoes a constant acceleration of -2.0 m/s^2 while rolling up the ramp.
   (a) Write equations for her position and velocity as functions of time, setting $x = 0$ and $t = 0$ when she is at the bottom of the ramp.
   (b) Determine the maximum distance the skateboarder travels up the ramp.
   (c) How long does it take the skateboarder to reach its maximum distance?
   (d) After reaching the maximum distance, the skateboarder will roll back down ramp with the same acceleration of -2.0 m/s^2. How long will it take her to reach the bottom of the ramp and how fast will she be moving when she gets there?

8. An object is dropped out of a window in a tall building. It hits the ground after 2.0 seconds. How high above the ground is the window? (The acceleration of a freely-falling object is 9.8 m/s^2 downward.)