1-D Equations of Motion for Constant Acceleration

The definitions of average velocity and average acceleration may be used, along with special conditions imposed when motion occurs with constant acceleration, to derive equations that describe the motion of an object at different times.

The definitions of average velocity and average acceleration are

$$v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \text{and} \qquad a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Two special conditions that exist when motion occurs with constant acceleration are

(1) $v_{av} = \frac{1}{2}(v_i + v_f)$ and (2) $a_{av} = a$

Exercise: Use the above four relationships to derive the equations of motion for constant acceleration:

(0) $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ (2) $v_f = v_i + a\,\Delta t$

(1) $\Delta x = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$ (3) $v_f{}^2 = v_i{}^2 + 2a\Delta x$

Frequently these will be written assuming $t_i = 0$ and $t_f = t$ so that $\Delta t = t_f - t_i = t_f = t.$.
Then these equations may be written:

(0) $x_f = x_i + \frac{1}{2}(v_i + v_f)t$ (2) $v_f = v_i + a\,t$

(1) $x_f = x_i + v_i\,t + \frac{1}{2}at^2$ (3) $v_f{}^2 = v_i{}^2 + 2a(x_f - x_i)$

Other notation frequently used in physics texts sets $x_f = x$, $x_i = x_0$, $v_f = v$ and $v_i = v_0$. Using this notation, x_0 and v_0 are the position and velocity at time $t_0 = 0$ and x and v are the position and velocity at some later time t.

Then the 1-D equations of motion for constant acceleration are written:

(0) $x = x_0 + \frac{1}{2}(v_0 + v)t$ (2) $v = v_0 + a\,t$

(1) $x = x_0 + v_0\,t + \frac{1}{2}at^2$ (3) $v^2 = v_0^2 + 2a(x - x_0)\cdot$

We can use these equations to solve kinematics problems algebraically.