Perpendicular Vector Displacements

Although these exercises use displacement vectors, the methods can be generalized to deal with any vectors as long as you remember that you can *only* add or subtract vectors with the same units (displacement with displacement; velocity with velocity; force with force; and so on).

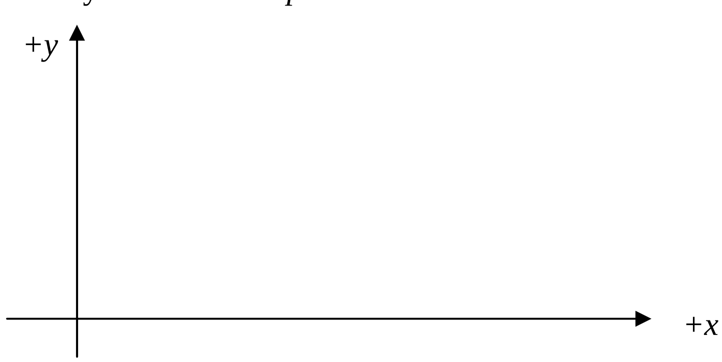
- **Form groups of 3 (or whatever number your instructor tells you) people.**
- **Each group will need**
  - **either a 1-meter stick, a 2-meter stick, or 10 m tape measure**
  - **a protractor**
  - **4 pieces of colored yarn (2 red, 1 yellow, and 1 orange)**
    Use red yarn for all *x*-displacements, yellow yarn for all *y*-displacements, and orange yarn for all *net* displacements.
    Note that the "net displacement" is from the starting point to the ending point for the entire walk.
  - **4 arrow heads cut from either scratch paper or cardboard.**

Exercise A: Two Horizontal Displacements That Are Perpendicular To Each Other

Choose an *x*-*y* coordinate system so that +*x* is horizontal and parallel to one wall and +*y* is horizontal and perpendicular to the chosen *x* axis. (Hopefully the second axis is parallel to another wall, but be sure to check that the two walls are actually at right angles.)

- Tape down one end of the *x*-displacement yarn and walk in the +*x* direction for 4 paces. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your first displacement, $\vec{\mathbf{D}}_1$.
- Now, tape down one end of the *y*-displacement yarn at the tip of $\vec{\mathbf{D}}_1$ and walk in the +*y* direction for 3 paces. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your second displacement, $\vec{\mathbf{D}}_2$.
- Finally, tape down one end of the *net* displacement yarn at the tail (beginning) of $\vec{\mathbf{D}}_1$ and walk from this initial position to your final position at the tip (end) of $\vec{\mathbf{D}}_2$. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your net displacement, $\vec{\mathbf{D}}_{net}$.

Sketch your displacement vectors below. A rough sketch will be fine for now. Use the *x* and *y* axes given. Is your sketch a *top view* or a *side view*?



Measure the lengths of all three vectors and the angles formed by the vectors. Record your results below:

Magnitude of $\vec{\mathbf{D}}_1$ (written $|\vec{\mathbf{D}}_1|$ or just D_1) = _______ m

Magnitude of $\vec{\mathbf{D}}_2$ (written $|\vec{\mathbf{D}}_2|$ or just D_2) = _______ m

Magnitude of $\vec{\mathbf{D}}_{\text{net}}$ (written $|\vec{\mathbf{D}}_{\text{net}}|$ or just D_{net}) = _______ m

Angle between $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_{\text{net}}$ = _______

Angle between $\vec{\mathbf{D}}_2$ and $\vec{\mathbf{D}}_{\text{net}}$ = _______

The following diagram and calculations are to be done on a separate piece of paper, before proceeding to Exercise B.

Scale Diagram:

Use a scale where 1 m is represented by 5 cm. Include this scale on your diagram by writing "5 cm represents 1 m".

Draw in and label *x* and *y* axes.

Use a ruler and protractor to make a careful diagram of your displacements.

Is your diagram a *top view* or a *side view*?

Calculations:

1. Check to see if $D_1 + D_2 = D_{\text{net}}$. (Note that this is the addition of the magnitudes without regard to the directions.) Is this what you would expect?

2. What relationship between the lengths do you expect to be true? Check to see if your measurements satisfy this relation.

3. Write the net displacement vector in terms of the components as follows:

$$\vec{\mathbf{D}}_{\text{net}} = D_1\,\hat{\mathbf{i}} + D_2\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{D}}_{\text{net}} = (\_\_\_\_\_\_\_\_\_\_\ \text{m})\,\hat{\mathbf{i}} + (\_\_\_\_\_\_\_\_\_\_\ \text{m})\,\hat{\mathbf{j}}$$

In vector notation, $\hat{\mathbf{i}}$ ("i-hat") represents the +*x*-direction; $\hat{\mathbf{j}}$, the +*y*-direction.

4. Check some ratios of the right triangle (that is, verify trig functions). Let α be the angle between $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_{\text{net}}$. Let β be the angle between $\vec{\mathbf{D}}_2$ and $\vec{\mathbf{D}}_{\text{net}}$.

   a. From your length measurements, calculate the ratio D_1/D_{net} and compare it with cos α and sin β. This is equivalent to verifying that $D_1 = D_{\text{net}} \cos\alpha = D_{\text{net}} \sin\beta$.

   b. Do a similar check for the ratio D_2/D_{net} and the corresponding trigonometric functions.

   c. Check that $\tan^{-1}(D_2/D_1) = \alpha$ and $\tan^{-1}(D_1/D_2) = \beta$.

Exercise B: One Horizontal Displacement And One Vertical Displacement

In Exercise B, the *x*-*y* coordinate system is chosen so that +*x* is horizontal and perpendicular to one wall and +*y* is vertically upward.

- Tape down one end of the *x*-displacement yarn 1.00 m away from a wall. Tape the other end of the yarn at the base of the wall. Place the tip of an arrowhead at that point showing the direction of the first displacement, $\vec{\mathbf{D}}_1$.
- Now, tape down one end of the *y*-displacement yarn at the tip of $\vec{\mathbf{D}}_1$. Tape the other end 1.50 m above the tip of $\vec{\mathbf{D}}_1$. Place the tip of an arrowhead at that point showing the direction of your second displacement, $\vec{\mathbf{D}}_2$.
- Finally, tape down one end of the *net* displacement yarn at the tail of $\vec{\mathbf{D}}_1$ and the other end at the tip of $\vec{\mathbf{D}}_2$. Place the tip of an arrowhead at that point showing the direction of your net displacement, $\vec{\mathbf{D}}_{net}$.

Sketch your displacement vectors below. A rough sketch will be fine for now. Be sure to include the *x* and *y* axes. Is your sketch a *top view* or a *side view*?

Measure the lengths of all three vectors and the angles formed by the vectors. Record your results below:

Magnitude of $\vec{\mathbf{D}}_1$ (written $|\vec{\mathbf{D}}_1|$ or just D_1) = _______ m

Magnitude of $\vec{\mathbf{D}}_2$ (written $|\vec{\mathbf{D}}_2|$ or just D_2) = _______ m

Magnitude of $\vec{\mathbf{D}}_{net}$ (written $|\vec{\mathbf{D}}_{net}|$ or just D_{net}) = _______ m

Angle between $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_{net}$ = _______

Angle between $\vec{\mathbf{D}}_2$ and $\vec{\mathbf{D}}_{net}$ = _______

Do a scale diagram as in Exercise A on a separate piece of paper. Do the same calculations as in Exercise A for Exercise B.

Exercise C: Three Horizontal Displacements

Use the *x*-*y* axes you chose in Exercise A.

- Tape down one end of an *x*-displacement yarn and walk in the +*x* direction for 3 paces. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your first displacement, $\vec{\mathbf{D}}_1$.
- Now, tape down one end of the *y*-displacement yarn at the tip of $\vec{\mathbf{D}}_1$ and walk in the +*y* direction for 2 paces. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your second displacement, $\vec{\mathbf{D}}_2$.
- Now, tape down one end of a second *x*-displacement yarn at the tip of $\vec{\mathbf{D}}_2$ and walk in the +*x* direction for 1 pace. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your third displacement, $\vec{\mathbf{D}}_3$.
- Finally, tape down one end of the *net* displacement yarn at the tail of $\vec{\mathbf{D}}_1$ and walk from this initial position to your final position at the tip of $\vec{\mathbf{D}}_3$. Let the yarn trail behind you as you walk. Tape the yarn down where you stop. Place the tip of an arrowhead at that point showing the direction of your net displacement, $\vec{\mathbf{D}}_{net}$.

Leave your displacement vectors on the floor for Exercise D.

Sketch your displacement vectors below. A rough sketch will be fine for now. Be sure to include the *x* and *y* axes. Is your sketch a *top view* or a *side view*?

Measure the lengths of all four vectors, the angle $\vec{\mathbf{D}}_{net}$ makes with the +*x* axis and the angle $\vec{\mathbf{D}}_{net}$ makes with the +*y* axis. Record your results below:

Magnitude of $\vec{\mathbf{D}}_1$ (written $|\vec{\mathbf{D}}_1|$ or just D_1) = _______ m
Magnitude of $\vec{\mathbf{D}}_2$ (written $|\vec{\mathbf{D}}_2|$ or just D_2) = _______ m
Magnitude of $\vec{\mathbf{D}}_3$ (written $|\vec{\mathbf{D}}_3|$ or just D_3) = _______ m
Magnitude of $\vec{\mathbf{D}}_{net}$ (written $|\vec{\mathbf{D}}_{net}|$ or just D_{net}) = _______ m

Angle between $\vec{\mathbf{D}}_{net}$ and the +*x* axis = _______ = α
Angle between $\vec{\mathbf{D}}_{net}$ and the +*y* axis = _______ = β

The following diagram and calculations are to be done on a separate piece of paper.

Scale Diagram:

Use a scale where 1 m is represented by 5 cm. Include this scale on your diagram by writing "5 cm represents 1 m".
Draw in and label *x* and *y* axes.
Use a ruler and protractor to make a careful diagram of your displacements.
Is your diagram a *top view* or a *side view*?

Calculations:

1. Check to see if $D_1 + D_2 + D_3 = D_{net}$. Is this what you would expect?
2. Knowing we can add the magnitudes of parallel vectors, find:

   Net *x* displacement = $D_{net,\,x}$ = ___________ m

   Net *y* displacement = $D_{net,\,y}$ = ___________ m

3. Write the net displacement vector in terms of the components as follows:

$$\vec{\mathbf{D}}_{net} = D_{net,\,x}\,\hat{\mathbf{i}} + D_{net,\,y}\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{D}}_{net} = (\_\_\_\_\_\_\_\_\_\_\_\ \text{m})\,\hat{\mathbf{i}} + (\_\_\_\_\_\_\_\_\_\_\_\ \text{m})\,\hat{\mathbf{j}}$$

4. Check that the magnitude of $\vec{\mathbf{D}}_{net}$ satisfies the Pythagorean theorem using the *x* and *y* components of $\vec{\mathbf{D}}_{net}$.
5. Check that $\tan^{-1}(D_y/D_x) = \alpha$ and $\tan^{-1}(D_x/D_y) = \beta$.

Exercise D: Three Horizontal Displacements (One in the negative *x*-direction)

Modify your vectors from Exercise C. Do not change $\vec{\mathbf{D}}_1$ or $\vec{\mathbf{D}}_2$. Change the direction (but not length!) of $\vec{\mathbf{D}}_3$ so that it is in the negative *x*-direction. Show the net displacement with yarn and an arrowhead.

Sketch your displacement vectors below. A rough sketch will be fine for now. Be sure to include the *x* and *y* axes. Is your sketch a *top view* or a *side view*?

Measure the length of $\vec{\mathbf{D}}_{\text{net}}$, the angle $\vec{\mathbf{D}}_{\text{net}}$ makes with the +*x* axis and the angle $\vec{\mathbf{D}}_{\text{net}}$ makes with the +*y* axis. The magnitudes of the other vectors should be the same as in Exercise C. Record your results below:

Magnitude of $\vec{\mathbf{D}}_1$ (written $|\vec{\mathbf{D}}_1|$ or just D_1) = _______ m
Magnitude of $\vec{\mathbf{D}}_2$ (written $|\vec{\mathbf{D}}_2|$ or just D_2) = _______ m
Magnitude of $\vec{\mathbf{D}}_3$ (written $|\vec{\mathbf{D}}_3|$ or just D_3) = _______ m
Magnitude of $\vec{\mathbf{D}}_{\text{net}}$ (written $|\vec{\mathbf{D}}_{\text{net}}|$ or just D_{net}) = _______ m

Angle between $\vec{\mathbf{D}}_{\text{net}}$ and the +*x* axis = _______ = α
Angle between $\vec{\mathbf{D}}_{\text{net}}$ and the +*y* axis = _______ = β

Do a scale diagram as in Exercise C on a separate piece of paper. Do the same calculations as in Exercise C for Exercise D.