More Graphs of Linear Equations

Linear equations are frequently written in the slope-intercept form: $y = mx + b$. This expresses y (the dependent variable) as a function of x (the independent variable). In this equation m is the slope of the line ($m = \Delta y/\Delta x$) and b is the y-intercept (the value of y when $x = 0$).

Given a linear graph and two points on the line, it is possible to determine the slope of the line and to use the point-slope form to write an equation for the line. This equation can then be rewritten in slope-intercept form.

Please do all exercises on a separate sheet of paper. Graphs should be done on graph paper.

Next we will look at some special cases of linear equations.

Special Case #1: If the y-intercept is zero ($b = 0$), then y is said to be directly proportional to x. The constant of proportionality is m, the slope of the corresponding graph of the line. Whenever the dependent variable is directly proportional to the independent variable the resulting graph will be a straight line passing through the origin.

Exercise #1: The mass of a piece of aluminum is directly proportional to its volume. The mass and volume of several pieces of aluminum are recorded in the table below.

a. Plot a graph of mass versus volume. Should you include the origin as a point on the graph? (Hint: What is the mass of a piece of aluminum with a volume of zero?)

b. Determine the slope of the graph. Include units. The slope is the proportionality constant which in this case is the density of aluminum.

c. Write an equation for the mass as a function of the volume.

Volume (cubic cm)	Mass (g)
1.00	2.75
1.50	4.00
2.00	5.40
2.50	6.75
3.00	8.15

Special Case #2: If the slope is zero ($m = 0$), then y is a constant ($y = b$). The value of y is independent of the value of x. This type of graph will not be of interest in your physics courses.

Special Case #3: If the slope of the graph is positive (m>0), y is a linearly increasing function of x. This means that when x increases, y also increases.

Exercise #3: Graph the equation y = 3x + 5.

Special Case #4: If the slope of the graph is negative (m<0), y is a linearly decreasing function of x. This means that when x increases, y decreases.

Exercise #4: Graph the equation y = -3x + 5.

Exercise #5: The height of the bottom of a hanging spring above a tabletop is measured as different weights are hung from the spring. This data is given in the table below.

a. Plot a graph of height above the tabletop as a function of the weight hung from the spring.

b. Calculate the slope of the resulting graph.

c. Write an equation relating the height of the bottom of the spring above the tabletop to the weight hung from the spring.

d. Is the height a linearly increasing or a linearly decreasing function of the weight?

e. According to your equation, what is the value of the height when no weight is hung from the spring? Does this agree with your graph?

f. Is the height directly proportional to the weight?

weight (lbs)	height (in)
1.00	36.00
1.50	34.25
2.00	32.50
2.50	31.00
3.00	29.15
3.50	27.50
4.00	25.75
4.50	24.10
5.00	22.25