

Mathematical Graphs Equations of a Line

Mathematical Graphs and Scientific Graphs

In mathematics, graphs are made while studying functions to give a feel for the shape of the graph of a function. A mathematical graph is often made of a function that is known to show how the function varies as an independent variable varies. Mathematical graphs are also used to show solutions to equalities and inequalities. Mathematical graphs will be discussed in more detail in the rest of this section.

In science, graphs are made to show relationships between physical quantities, for example the relationship between the density of water and its temperature. A scientific graph is often made when a function is unknown and an experiment has been performed to determine the dependence of one quantity on another. Scientific graphs will be discussed in more detail in a later section.

Mathematical Graphs

Some examples of functions that might be represented graphically are $f(x) = 2x + 3$, $g(x) = \sin(x)$, and $h(x) = 6e^x$. A mathematical graph of a function is created by choosing values for the independent variable, substituting each value into the given function and evaluating. These exact solutions to a given function are plotted and a line or curve is drawn that passes through each of the points.

When solving equations or systems of equations, graphs can be used to display solutions visually. If asked to graph the solutions to an equation such as $2v + 3t = 12$, we find ordered pair solutions and plot them on a graph. To do this we can choose a value for either variable and solve for the other variable. If $v = 0$, then $3t = 12$ or $t = 4$. If $t = 0$, then $2v = 12$ or $v = 6$. Since we know from experience that the solutions should lie along a line, these two points will allow us to draw a line on a set of v and t axes showing all of the solutions. Curves are more difficult, but we can show on a graph the solutions to $3x^2 + 4y^2 = 6$ which is an ellipse.

Graphs of Linear Functions

The following are examples of linear functions: $y = 3x - 2$, $k = \frac{3}{2}c + 2$, and $v = 10t + \frac{7}{3}$. These examples are all written as equations in the slope-intercept form. In these equations, the variable on the right-hand side of the equation is the independent variable while the variable on the left-hand side of the equation is the dependent variable.

$y = 3x - 2$ is in the form of $y = mx + b$, where the slope is $m = 3$ and the y -intercept is $b = -2$.

$k = \frac{3}{2}c + 2$ is in the form of $k = mc + b$, where the slope is $\frac{3}{2}$ and the k -intercept is 2.

$v = 10t + \frac{7}{3}$ is in the form of $v = mt + b$, where the slope is 10 and the v -intercept is $\frac{7}{3}$.

Very often in math classes the independent variable is chosen to be x and the dependent variable is chosen to be y . Making this choice should generally be avoided in science. For a mathematical graph, the slope and intercept of a line are often reported as fractions as in the last two examples above.

One method of plotting the graph of a linear equation is simply to choose some values for the independent variable and use these to calculate the corresponding values of the dependent variable. The ordered pairs that result will then be plotted on a graph. The line that passes through these points will be drawn with a straight edge.

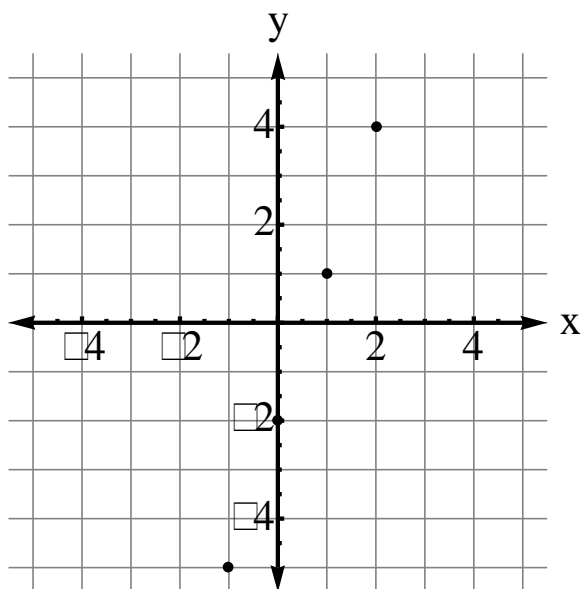
Example 1: Plot the graph of the equation $y = 3x - 2$,

First construct a table of values: Each y value is calculated from a chosen x value. The first row is filled in by setting $y = 3(0) - 2$ and simplifying to get $y = -2$. The rest of the table has been filled out in a similar fashion.

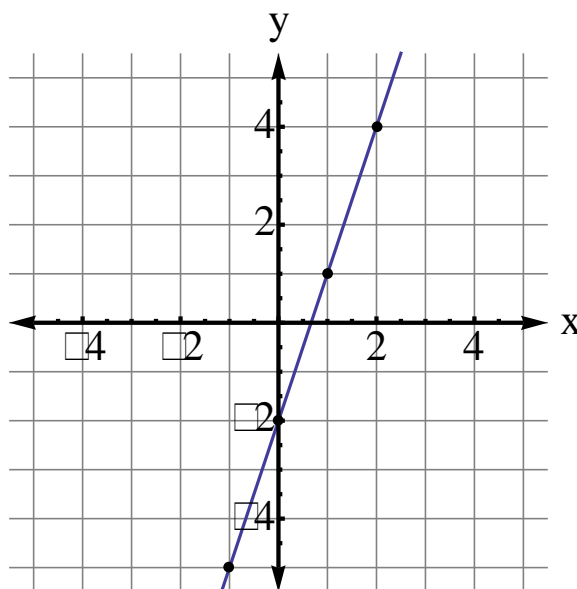
X	y
0	-2
1	1
-1	-5
2	4

Then plot these points on the grid provided below. Finally, draw in the line that passes through these points.

(with points added)



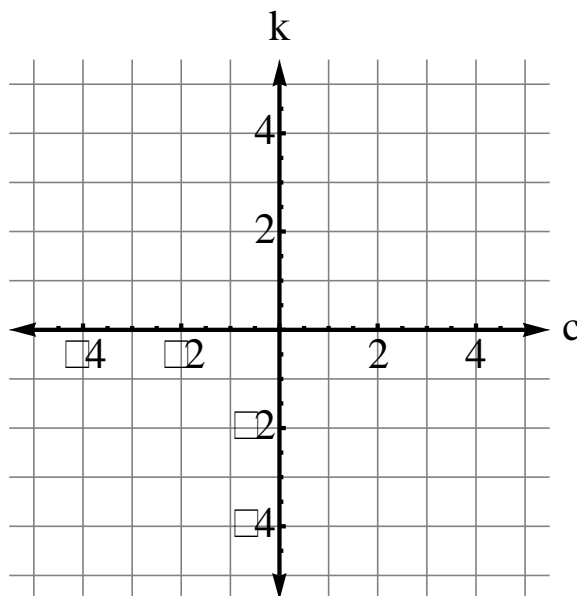
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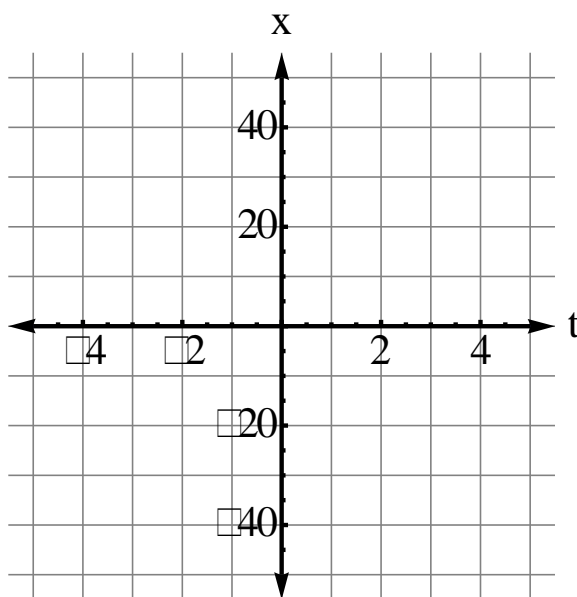
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Exercise 1: Graph the linear function $k = \frac{3}{2}c + 2$. Construct a table of values as in the example and then plot these points on the provided grid. Draw in the line that passes through these points. Check your line by using the k-intercept and the slope.

c	k
0	
2	
-2	
4	
-4	



Exercise 2: On the axes given below, plot the solutions to the equation: $x = 15t + 5$. Include a table with at least 3 ordered pairs of values.



In some cases, we may be given a mathematical graph of a linear function and need to find the slope and write an equation. In this case, we will use two points on the graph to calculate the slope of the graph and to write an equation for the line.

To find the slope, m , of a linear graph we will use the following definition:

$$\text{Slope} = \text{rise/run}$$

$$= \text{change in dependent variable/change in independent variable}$$

$$= \text{change in quantity on vertical axis/change in quantity on horizontal axis}$$

If a given graph plots y as a linear function of x and we have two points, (x_1, y_1) and (x_2, y_2) , these become

$$m = \Delta y / \Delta x = (y_2 - y_1) / (x_2 - x_1) = (y_1 - y_2) / (x_1 - x_2).$$

If (x_1, y_1) and (x_2, y_2) have known values, then we will have a numeric value for the slope. Once the slope has been calculated, either of the two points together with an arbitrary point (x, y) can be used to write an equation for the line in point-slope form.

To do this, we start with the definition of slope to write either

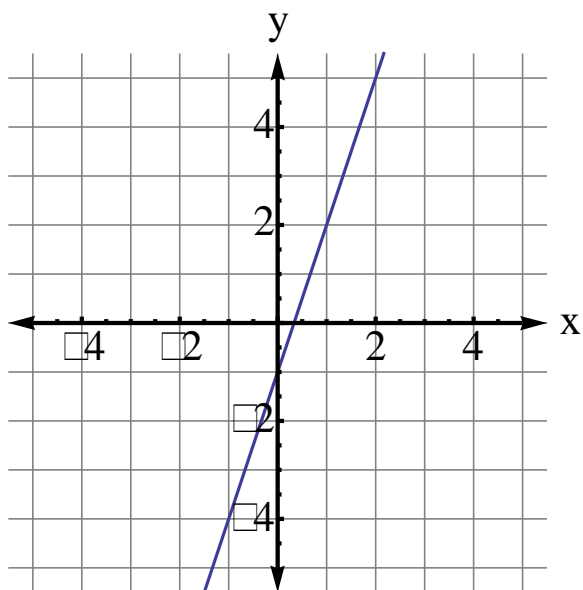
$$m = (y - y_1) / (x - x_1) \text{ or } m = (y - y_2) / (x - x_2).$$

These can be rewritten as

$$y - y_1 = m(x - x_1) \text{ or } y - y_2 = m(x - x_2),$$

which are in the point-slope form of the equation for a line.

Example 2: Find the slope of the graph given below. Write two equations for the line in point-slope form. Rewrite each of these two equations to obtain equations in slope-intercept form. Do these two equations agree? Should they?



Two points on the line are $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-1, -4)$. Using these points, we can calculate the slope of the graph

$$m = \Delta y / \Delta x = (y_2 - y_1) / (x_2 - x_1) = (-4 - 2) / (-1 - 1) \\ = -6 / -2 = 3.$$

Using the first point we get the equation

$$y - 2 = 3(x - 1).$$

Using the second point we get the equation

$$y - (-4) = 3[x - (-1)] \text{ or } y + 4 = 3(x + 1).$$

Simplify the first equation by first using the distributive property: $y - 2 = 3x - 3$

Solve for y (by adding 2 to each side of the equation) to get the equation in slope-intercept form: $y = 3x - 1$.

Applying similar steps to the second equation yields $y = 3x - 1$.

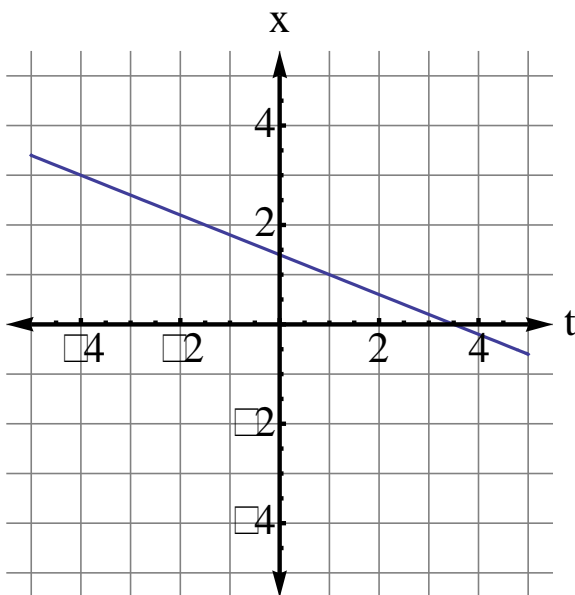
The two equations agree, as expected.

The slope-intercept equation for the line is $y = 3x - 1$. From this equation, the slope is $m = 3$ and the y -intercept is $b = -1$. Inspection of the graph shows that these are the correct results.

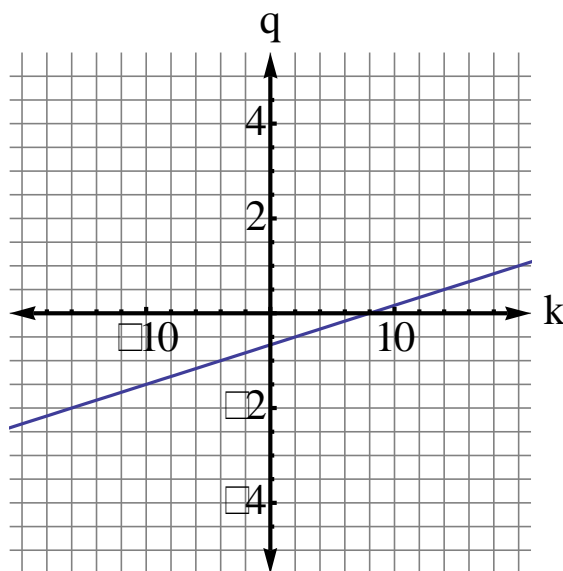
Exercise 3: To be done on separate paper. For each graph given below:

- Determine the slope of the line.
- Write two equations for the line in point-slope form.
- Rewrite each of these two equations in slope-intercept form. If your equations do not agree, check your work.
- Check that the intercept you found is in qualitative agreement with the graph.

a)



b)



Exercise 4: Draw in and label axes on the grid below:

Choose an appropriate scale and plot the solutions to the equation: $30x - 8y = 120$.
Determine the slope of the graph and the x and y intercepts.

