Mathematical Graphs
Equations of a Line

Mathematical Graphs and Scientific Graphs

In mathematics, graphs are made while studying functions to give a feel for the shape of the graph of a function. A mathematical graph is often made of a function that is known to show how the function varies as an independent variable varies. Mathematical graphs are also used to show solutions to equalities and inequalities. Mathematical graphs will be discussed in more detail in the rest of this section.

In science, graphs are made to show relationships between physical quantities, for example the relationship between the density of water and its temperature. A scientific graph is often made when a function is unknown and an experiment has been performed to determine the dependence of one quantity on another. Scientific graphs will be discussed in more detail in a later section.

Mathematical Graphs

Some examples of functions that might be represented graphically are $f(x) = 2x + 3$, $g(x) = \sin(x)$, and $h(x) = 6e^x$. A mathematical graph of a function is created by choosing values for the independent variable, substituting each value into the given function and evaluating. These exact solutions to a given function are plotted and a line or curve is drawn that passes through each of the points.

When solving equations or systems of equations, graphs can be used to display solutions visually. If asked to graph the solutions to an equation such as $2v + 3t = 12$, we find ordered pair solutions and plot them on a graph. To do this we can choose a value for either variable and solve for the other variable. If $v = 0$, then $3t = 12$ or $t = 4$. If $t = 0$, then $2v = 12$ or $v = 6$. Since we know from experience that the solutions should lie along a line, these two points will allow us to draw a line on a set of v and t axes showing all of the solutions. Curves are more difficult, but we can show on a graph the solutions to $3x^2 + 4y^2 = 6$ which is an ellipse.

Graphs of Linear Functions

The following are examples of linear functions: $y = 3x – 2$, $k = \frac{3}{2}c + 2$, and $v = 10t + \frac{7}{3}$. These examples are all written as equations in the slope-intercept form. In these equations, the variable on the right-hand side of the equation is the independent variable while the variable on the left-hand side of the equation is the dependent variable.

$y = 3x – 2$ is in the form of $y = mx + b$, where the slope is $m = 3$ and the y-intercept is $b = –2$.

$k = \frac{3}{2}c + 2$ is in the form of $k = mc + b$, where the slope is $\frac{3}{2}$ and the k-intercept is 2.

$v = 10t + \frac{7}{3}$ is in the form of $v = mt + b$, where the slope is 10 and the v-intercept is $\frac{7}{3}$.

Very often in math classes the independent variable is chosen to be x and the dependent variable is chosen to be y. Making this choice should generally be avoided in science. For a mathematical graph, the slope and intercept of a line are often reported as fractions as in the last two examples above.

One method of plotting the graph of a linear equation is simply to choose some values for the independent variable and use these to calculate the corresponding values of the dependent variable. The ordered pairs that result will then be plotted on a graph. The line that passes through these points will be drawn with a straight edge.

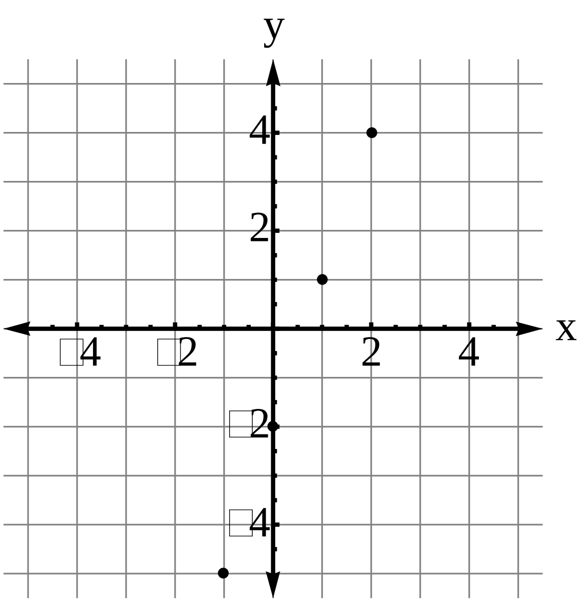
Example 1: Plot the graph of the equation y = 3x – 2,

First construct a table of values: Each y value is calculated from a chosen x value. The first row is filled in by setting y = 3(0) – 2 and simplifying to get y = -2. The rest of the table has been filled out in a similar fashion.

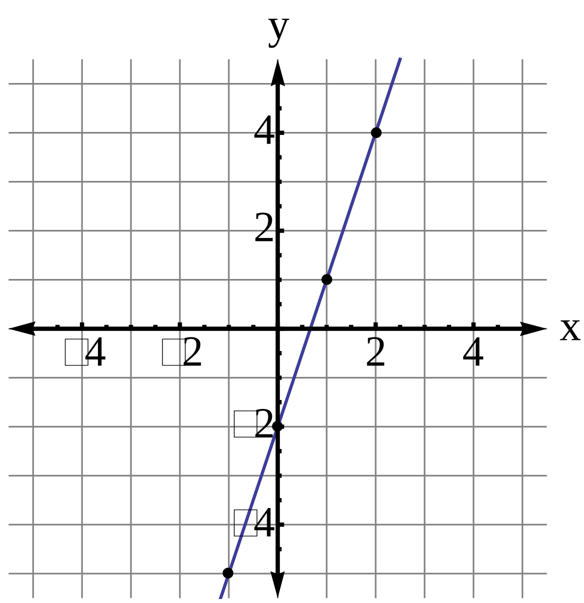
X	y
0	-2
1	1
-1	-5
2	4

Then plot these points on the grid provided below. Finally, draw in the line that passes through these points.

(with points added)



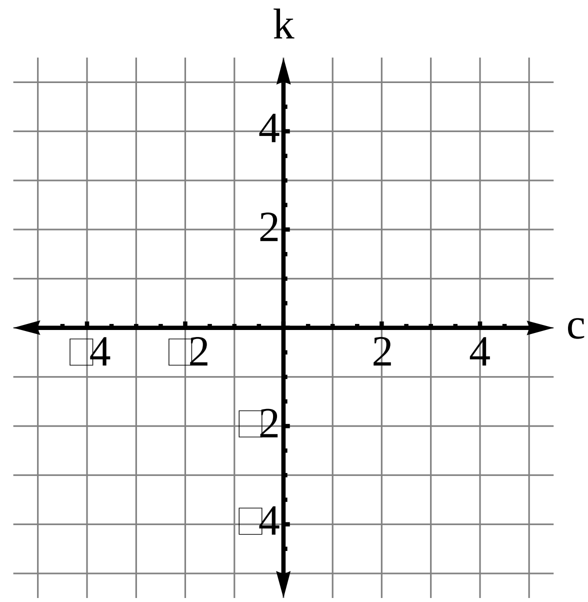
(with line added)



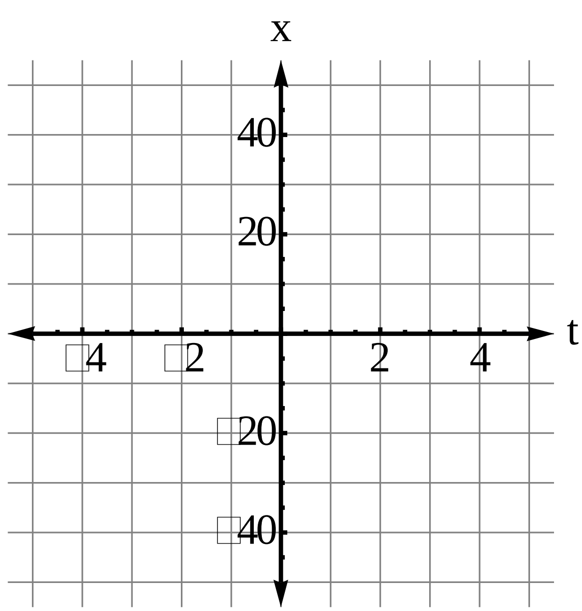
Name: ________________________

Exercise 1: Graph the linear function k = $\frac{3}{2}$c + 2. Construct a table of values as in the example and then plot these points on the provided grid. Draw in the line that passes through these points. Check your line by using the k-intercept and the slope.

c	k
0	
2	
-2	
4	
-4	



Exercise 2: On the axes given below, plot the solutions to the equation: x = 15t + 5. Include a table with at least 3 ordered pairs of values.



In some cases, we may be given a mathematical graph of a linear function and need to find the slope and write an equation. In this case, we will use two points on the graph to calculate the slope of the graph and to write an equation for the line.

To find the slope, m, of a linear graph we will use the following definition:

Slope = rise/run

= change in dependent variable/change in independent variable

= change in quantity on vertical axis/change in quantity on horizontal axis

If a given graph plots y as a linear function of x and we have two points, (x_1, y_1) and (x_2, y_2), these become

$$m = \Delta y/\Delta x = (y_2 - y_1)/(x_2 - x_1) = (y_1 - y_2)/(x_1 - x_2).$$

If (x_1, y_1) and (x_2, y_2) have known values, then we will have a numeric value for the slope. Once the slope has been calculated, either of the two points together with an arbitrary point (x, y) can be used to write an equation for the line in point-slope form.

To do this, we start with the definition of slope to write either

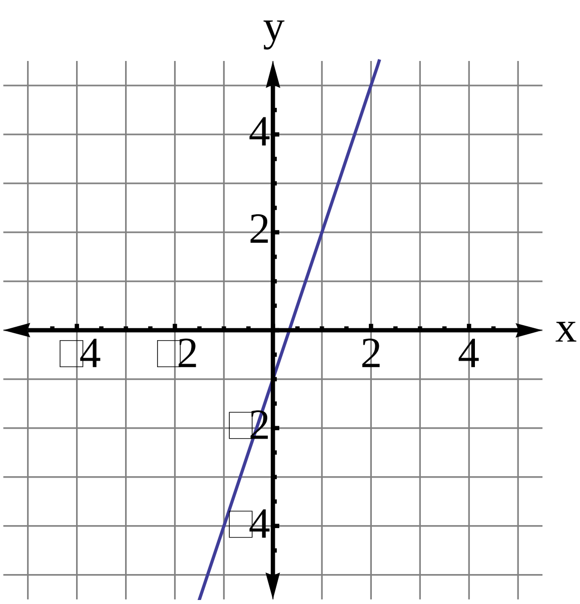
$$m = (y - y_1)/(x - x_1) \text{ or } m = (y - y_2)/(x - x_2).$$

These can be rewritten as

$$y - y_1 = m(x - x_1) \text{ or } y - y_2 = m(x - x_2),$$

which are in the point-slope form of the equation for a line.

Example 2: Find the slope of the graph given below. Write two equations for the line in point-slope form. Rewrite each of these two equations to obtain equations in slope-intercept form. Do these two equations agree? Should they?



Two points on the line are $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-1, -4)$. Using these points, we can calculate the slope of the graph

$m = \Delta y/\Delta x = (y_2 - y_1)/(x_2 - x_1) = (-4 - 2)/(-1 - 1)$
$= -6/-2 = 3.$

Using the first point we get the equation

$y - 2 = 3(x - 1).$

Using the second point we get the equation

$y - (-4) = 3[x - (-1)]$ or $y + 4 = 3(x + 1).$

Simplify the first equation by first using the distributive property: $y - 2 = 3x - 3$

Solve for y (by adding 2 to each side of the equation) to get the equation in slope-intercept form: $y = 3x - 1$.

Applying similar steps to the second equation yields

$y = 3x - 1.$

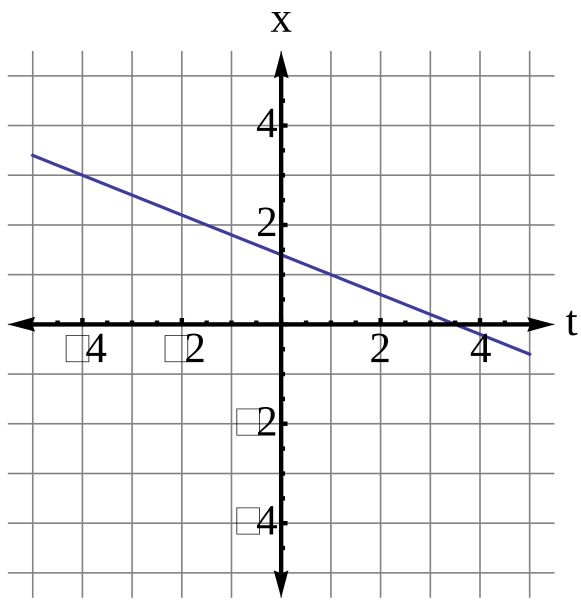
The two equations agree, as expected.

The slope-intercept equation for the line is $y = 3x - 1$. From this equation, the slope is $m = 3$ and the y-intercept is $b = -1$. Inspection of the graph shows that these are the correct results.

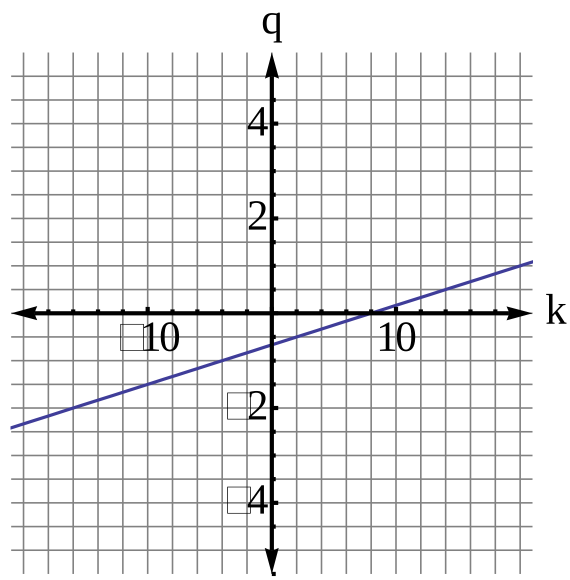
Exercise 3: To be done on separate paper. For each graph given below:

- Determine the slope of the line.
- Write two equations for the line in point-slope form.
- Rewrite each of these two equations in slope-intercept form. If your equations do not agree, check your work.
- Check that the intercept you found is in qualitative agreement with the graph.

a)

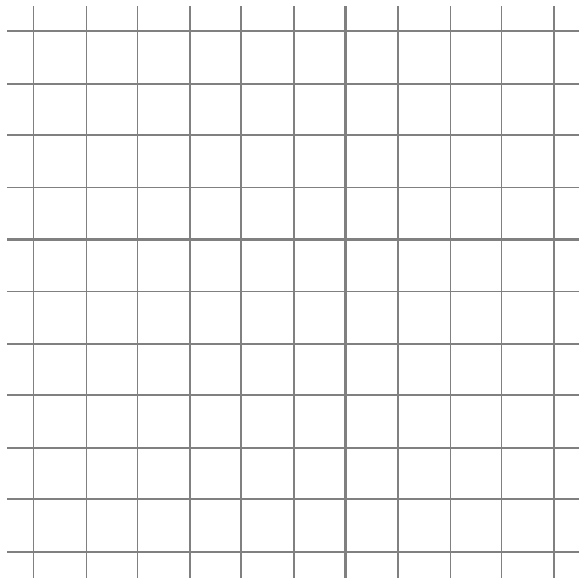


b)



Exercise 4: Draw in and label axes on the grid below:

Choose an appropriate scale and plot the solutions to the equation: 30 x – 8 y = 120. Determine the slope of the graph and the x and y intercepts.



Scientific Graphs
Equations of a Line

Mathematical Graphs and Scientific Graphs

In mathematics, graphs are made while studying functions to give a feel for the shape of the graph of a function. Mathematical graphs have been discussed in detail in the previous section.

In science, graphs are made to show relationships between physical quantities, for example the relationship between the density of water and its temperature. A scientific graph is often made when a relationship is unknown and an experiment has been performed to determine the dependence of one quantity on another. Scientific graphs will be discussed in more detail in this section.

Scientific Graphs

Scientific graphs are usually based on an experiment carried out to determine the relationship between two physical quantities when one is expected to depend on the other. You may know that the density of water depends on its temperature and you may perform an experiment that allows you to measure the density of water at different temperatures. By plotting your data on a graph you may be able to determine the relationship between the density and temperature.

You may find that the density varies linearly with temperature or that the density varies parabolically with the temperature. Often, the plotted points of the data are examined and a functional relationship is inferred. In the case of a linear relationship, a "best-fit" line is drawn with a straightedge that ***comes close*** to all of the points. **It is never expected that the line pass through any of the points due to experimental error.**

Graphing and Interpreting Scientific Data

When finding the equation relating the dependent variable to the independent variable on a scientific graph, letters should be used which represent the quantities graphed. For our example of a density vs. temperature graph for water, T would be a good independent variable for temperature and ρ would be a good dependent variable for density. (In physics, ρ is used for density, because d is commonly diameter which may be involved in a calculation of density.)

Some general rules for graphing you should follow are:

- Graphs should always be done on graph paper and should take up most of the space on the page.
- When graphing data the independent variable is plotted on the horizontal axis and the dependent variable is plotted on the vertical axis. An appropriate range and scale must be chosen for each axis.
- Each axis should be clearly labeled with the name and/or symbol of the quantity being plotted along with the appropriate units.
- Graphs should always have a descriptive title summarizing what the graph represents.

For our example, if a linear relationship between density and temperature is found, one will want to find the slope and an equation for the "best-fit" line $\rho = mT + b$.

The slope of a scientific graph is defined the same way as the slope for a mathematical graph, that is the change in the dependent variable divided by the change in the independent variable. Units must always be included when calculating or stating the slope of a scientific graph.

The slope of a linear graph of density versus temperature would be calculated using $slope = m = \frac{\Delta\rho}{\Delta T}$. In the metric system, density is measured in kg/m^3 and temperature in °C or K (Kelvin). The slope in our example would then have units of $kg/(m^3\ °C)$. The intercepts will also have units.

To calculate the slope of the "best-fit" line on a scientific graph one must choose two points on the ***line*** that are very far apart. If the graph has been made with a good scale on graph paper with at least 10 squares per inch, one should always be able to choose the points so that the change in each variable has ***three significant figures***. **Data points should never be used to calculate the slope of a "best-fit" line *even if* they seem to be on the line.** If you misplot 1 or 2 data points on a graph with 8 or 10 data points, the misplotted points will have very little effect on the "best-fit" line you draw. But, if you happen to use the value of a misplotted data point in the calculation of a slope, it can have a very large effect.

Given two points (T_1, ρ_1) and (T_2, ρ_2), the slope is $m = \frac{\rho_1 - \rho_2}{T_1 - T_2}$. An equation can be quickly written for the line in the point-slope form $(\rho - \rho_1) = m\,(T - T_1)$ or $(\rho - \rho_2) = m\,(T - T_2)$, using either point.

For a scientific graph, the slope and intercept of a line are ***never*** reported as fractions. They are always reported in decimal notation or scientific notation and ***always*** include units.

The correct procedure for graphing and interpreting scientific data will be developed in class for our example by providing a set of possible experimental values of temperature and density for a sample of water.

Temperature (°C)	Density (kg/m^3)
37.8	993.8
40.5	992.2
46.0	988.6
57.3	983.8
60.3	982.0
62.6	980.9
73.8	975.3
79.4	972.7

Exercise 1. For the graph made in class for the density of water, calculate the slope to three significant figures and be sure to include units.

Exercise 2. Use the point-slope equation and write down two equations for the line in the graph for the density of water (each equation using one of the two points you chose to calculate the slope).

Exercise 3. Put the equations you found in Exercise 2 into slope-intercept form. Are the intercepts in close agreement? Do they agree with the intercept seen on the graph?

Horizontal Motion of a Rolling Ball
Measurement and Graphing

In this exercise, you and your partners will record the motion of a bowling ball rolling on a smooth horizontal surface. This data will then be graphed and analyzed.

To perform this exercise, each group will need: a ball, a stopwatch for all but two group members, a 30-meter measuring tape and a roll of blue masking tape.

Procedure:

1. Using the measuring tape and the blue masking tape make a number line on the floor. Pieces of tape should be placed every 2.00 meters from 0.00 m to 8.00 m.
2. Assign one member of the group to roll the ball and one to stop the ball. The other group members will each be assigned a position along the number line. They will record the time for the ball to travel each distance. The timers will all start their stopwatches when the ball passes the zero mark. They will stop their stopwatches as the ball rolls past their assigned distance. Practice until all timers at a particular distance have reasonably close times. (What are reasonably close times?) You should include the time for the ball to roll a distance of zero meters in your data table.
3. Record one set of data for the ball being rolled at a slower speed and one set of data for the ball rolled at a faster speed.

	Slow Ball	Fast Ball
Distance (m)	Time (s)	Time (s)

Plot your data points on graph paper. Graph both sets of data on the same graph. Use different symbols and/or different colors to distinguish the data for the slow ball from the data for the fast ball.

Recall, the general rules for graphing you should follow are:

Graphs should always be done on graph paper and should take up most of the space on the page. An appropriate range and scale must be chosen for each axis.

Graphs should always have a descriptive title summarizing what the graph represents.

Each axis should be clearly labeled with the name or symbol of the quantity being plotted along with the appropriate units.

When graphing data the independent variable is plotted on the horizontal axis and the dependent variable is plotted on the vertical axis.

Your data table has a column for distance and a column for time to travel this distance for each ball. Which is the independent variable and which is the dependent variable?

Once you have determined which information is to be plotted on each axis, you must determine the best way to set up your graph. Some questions to be asked include:

What is an appropriate title for the graph?

Should the horizontal axis or the vertical axis be longer?

What is an appropriate range for each axis?

What is an appropriate scale for each axis?

You will be using this graph to predict the time to travel 10 meters, so your range and scale should allow for this. It is often a very good idea to know exactly how a graph will be used before actually creating a graph.

What type of smooth curve seems to best fit your data points? (Remember that, mathematically speaking, a straight line is a curve.)

If effects of friction and non-level floors are ignored, the data is expected to be linear. Use a ruler to draw in a best-fit line

How does the distance traveled for each case depend on the elapsed time? (Does the distance increase or decrease with time? Does the distance seem to be changing uniformly with time?).

Discuss any differences in the graphs for the slow and fast balls.

From your graphs, estimate the time it would take for each ball to travel 10 meters.

From your graphs, determine the distance traveled by each ball in the first second.

From your graph, determine the distance traveled by each ball in the second second. (that is, the distance traveled between $t = 1s$ and $t = 2s$)

From your graphs, determine the distance traveled by each ball in the third second.

Does the distance traveled by each ball in one second depend on which one second interval is chosen?

Horizontal Motion of a Rolling Ball
Analyzing the Graphs

The following exercises will be done after your graphs have been checked and returned to you by your instructor.

Exercise 1. Calculate the slope of the two lines on your graph of the bowling ball rolling in the hallway to three significant figures. Circle the two points on each line that you are using to calculate each slope. **DO NOT mark through the two points. They should still be readable.**

Exercise 2. Use the point-slope equation and write down two equations for each line on your graph of the bowling ball rolling in the hallway (each equation using one of the two points you chose to calculate the slope).

Exercise 3. Put the equations you found in Exercise 2 into slope-intercept form. Do the intercepts in the equations agree with the intercepts seen on the graph?

Group ______________ Name __________________________

Motion of a Ball Rolling Down an Incline
Measurement and Graphing

In this exercise, you and your partners will record the motion of a ball rolling on a smooth inclined ramp. This data will then be graphed and analyzed.

To perform this exercise, you will need: a ball, stopwatches, a board with marked distances, and something to support one end of the board.

Procedure:

1. Support one end of the plank approximately 15 to 20 centimeters above the lab table. This can be done using an equipment box or a stack of books. The exact height does not matter.
2. One member of the group will release the ball from the zero centimeter mark, the others will record the time for the ball to travel each distance (20 cm, 50 cm, 90 cm, 150 cm, and 225 cm). The timers will all start their stopwatches when the ball is released and stop them when it rolls past their particular distance.
3. Record at least five sets of data in Data Table 1. If at least four of the times at each distance are reasonably close, use the average of these four times as the time to roll that distance. (What are reasonably close times?) If you do not have four reasonably close times you should continue recording more data until you have enough close data points to feel confident which values should be averaged.
4. Record the average times for each distance in Data Table 2. You should include the time for the ball to roll a distance of zero centimeters in your data table.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8
Distance (cm)	Time(s)	Time(s)	Time(s)	Time(s)	Time(s)	Time(s)	Time(s)	Time(s)
20.0								
50.0								
90.0								
150.0								
225.0								

Data Table 2

Trials Used to Calculate Average Time ______________________________

Distance (cm)	Average Time (s)
0	
20	
50	
90	
150	
225	

Plot your data points on graph paper. Follow the rules of graphing as discussed previously. Consider time to be the independent variable.

Scale graph axes so data covers a large portion of the graph.
Be sure graph will allow you to answer the questions below.

Draw in a smooth curve that best fits your data points. Does this look like the type of smooth curve that best fit the data for the ball rolling on a horizontal surface?

How does the distance traveled for each case depend on the elapsed time? (Does the distance increase or decrease with time? Does the distance seem to be changing uniformly with time?)

From your graph, estimate the time it took for the ball to travel 100 centimeters.

From your graph, determine the distance traveled by the ball in the first second.

From your graph, determine the distance traveled by the ball in the second second.

Does the distance traveled in one second depend on which one second interval is chosen?

Graphing and Predicting – Overhanging Blocks #2

Objective: Stack blocks on top of each other so that the top block extends out horizontally as far as possible without tipping the stack. Make a quantitative prediction as to how large the overhang can be for a given number of blocks.

Obtain 6 type "C" blocks, a meter stick and a straight edge.

1. Using just two blocks. Measure the largest overhang you can obtain. Record your measurement in the table below.
2. Using three blocks determine the largest overhang you can obtain. Again, measure the overhang and record it in the table.
3. Repeat the above step for 4, 5 and 6 blocks.

Number of Blocks	Maximum Overhang Obtained (cm)
2	
3	
4	
5	
6	

4. Plot a graph showing the data you have obtained. Some general rules for graphing are:

   Graphs should always be done on graph paper and should take up most of the space on the page.

   Graphs should always have a title summarizing what the graph represents.

   Each axis should be clearly labeled with the name or symbol of the quantity being plotted along with the appropriate units.

   When graphing data the independent variable is plotted on the horizontal axis and the dependent variable is plotted on the vertical axis. Your data table has two columns: a column of number of blocks and a column of maximum overhang obtained. Which of these is the independent variable and which is the dependent variable?

   Once you have determined which information is to be plotted on each axis, you must determine the best way to set up your graph. Some questions to be asked include:

   What is an appropriate title for the graph?

   Should the horizontal axis or the vertical axis be longer?

   What is an appropriate range for each axis?

   What is an appropriate scale for each axis?

   You will be using this graph to predict the maximum overhang possible for 7 and 8 blocks, so make sure these are included within the range of number of block values. It is often a very good idea to know exactly how a graph will be used before actually creating a graph.

When graphing, data it is important to be aware that all data contains some random error. It is therefore never useful to connect successive data points with straight line segments. It is best to draw a smooth curve that comes near or passes through each data point.

5. Draw a smooth curve that passes through or near each of your data points. Use your smooth curve to predict the maximum overhang possible using 7 blocks. Also predict the maximum overhang possible using 8 blocks. Since you are making a scientific prediction, you should be extending your curve by ***continuing the trend***. You should not first ***guess*** values for 7 and 8 blocks and draw a curve that supports your guess.

Number of Blocks	Predicted Maximum Overhang (cm)
7	
8	

6. Test your predictions by repeating step 2 above for 7 and 8 blocks. And record your experimental overhang.

Number of Blocks	Maximum Overhang Obtained Experimentally (cm)
7	
8	

7. Were you able to reach the predicted overhang? Were you able to exceed the predicted overhang?

Building Physical Intuition – Measurement 3 and Graphing

Groups: form groups of three
Equipment: each group should have a one meter stick, meter stick caliper jaws, Vernier caliper and several wooden cylinders.

Measurements

1. Use the meter stick and meter stick caliper jaws to measure the **length** of each cylinder. Record your measurements in the table below to the nearest 0.1 cm.
2. Use the vernier caliper to measure the **diameter** of each cylinder. Record your measurements in the table below to the nearest 0.01 cm.
3. Use a balance to measure the **mass** of each cylinder. Record your measurement in the table below to the nearest 0.01 g.

Length (cm)	Diameter (cm)	Mass (g)

Calculations

1. Plot a graph showing the mass and length of the cylinders which have the same diameter (or approximately the same diameter).
   Looking at the plotted points, what type of smooth curve will fit the plotted points?

   Does this make physical sense? How should the mass of two cylinders compare, if both have the same diameter and one is twice as long as the other?

2. Plot a second graph showing the mass and diameter of the cylinders that have the same length (or approximately the same length).
   Looking at the plotted points, will the same type of curve as the previous graph fit the plotted points?

   Does this make physical sense? How should the mass of two cylinders compare, if they have equal length and one has twice the diameter of the other?

3. Plot a third graph showing the mass and diameter squared of the cylinders that have the same length (or approximately the same length).
   Looking at the plotted points, what type of smooth curve will fit the plotted points?

   Does this make physical sense? How should the mass of two cylinders compare, if they have equal length and one has twice the diameter of the other?

4. Can you make a graph using data from all of your cylinders and obtain a straight line? Think about what quantity you might graph along with mass that should give a straight line.

   Make a graph and see if you get a straight line.

Your instructor may ask you to do some calculations after checking your graphs.

The Simple Pendulum

Purpose

In this exercise we will attempt to verify the relationship between the length and the period of a simple pendulum.

Introduction

A simple pendulum consists of a small object called a bob suspended by a light string. The bob being small means its size is small compared to the length of the string. The string being light means it has a mass much smaller than that of the bob. The period is defined to be the time for one oscillation, one back and forth motion.

The period of a simple pendulum for small amplitudes is $T = 2\pi\sqrt{\frac{L}{g}}$.

In this formula, L is the length of the pendulum measured from the pivot point to the center of mass of the bob and g is the acceleration of a freely-falling object (9.80 m/s^2 on Earth).

If we square both sides of the equation, $T^2 = 4\pi^2 \frac{L}{g} = \frac{4\pi^2}{g} L$.

We find a linear relationship between the square of the period and the length of the pendulum. We will find the period of 5 pendula each with a different length, but using the same bob. We will attempt to verify the relation by graphing our results.

Equipment

Each group will have:

A table clamp	A two-way clamp
A threaded rod	A hooked mass to be used as the bob
5 lengths of string	2 meterstick caliper jaws
A meterstick	A stopwatch
A sheet of paper to record data	

Procedure

Since the length of the pendulum is defined to be from the pivot point to the center of mass of the bob, we need to determine the location of the center of mass of the bob. Place the bob on the meterstick with its hooked end extended past the 0 cm mark. Slowly move the bob so that more and more of it is extended off the meterstick. Record the location of the bottom of the bob when it just begins to tip. This will be the distance the center of mass is above the bottom of the bob.

Make a table with 3 columns. The first column will be the distance of the bottom of the bob from the pivot point for each pendulum. The other two columns will be the time for 20 oscillations for each pendulum.

Attach the table clamp to the lab bench. Attach the two-way clamp and threaded rod to the vertical rod of the table clamp. Attach one of your strings to the threaded end of the rod. Hang the bob from the other end.

Move the bob so that the string makes an angle of about 10° with the vertical and release. Time 20 back and forth oscillations of the pendulum. Make sure you wait for one complete back and forth motion before counting "1". Repeat the process to obtain a second timing of 20 oscillations. If your two times are off by more than 0.5 s, perform a third trial. If none of the three times are within 0.5 s of each other, inform your instructor.

Using the meterstick and caliper jaws, measure the distance from the pivot point to the bottom of the bob. The location of the pivot point will depend on how you have the string attached to the rod.

Repeat the process using each of the other four strings.

Have your instructor check your data for reasonableness.

Analysis

Answer each question on separate paper.

Create a second table containing three columns. The first column should be the "Pendulum Length (cm)"; the second column, "Period (s)"; the third column, "Period Squared (s^2)".

Use your data to fill the table with the appropriate values.

Make a graph showing how the square of the period is related to the length of the pendulum. Choose a scale that will allow you to calculate a slope to three significant figures. Which variable belongs on the horizontal axis?

Does your data appear to fall along a straight line?

Regardless of your answer to the previous question, draw the straight line which best fits your data. The line should come close to all data points.

Determine the slope of the line you have drawn to three significant figures. And write an equation for your line.

From the equation, $T^2 = \frac{4\pi^2}{g}L$, the theoretical value for the slope of the line is

$$4\pi^2/(9.80\ \mathrm{m/s^2}) = 4.03\ \mathrm{s^2/m}$$

Compute the percent error in your slope based on this value.

From your graph or the equation for your line determine the length of a simple pendulum that will have a period of 1.00 s. Give your answer to the nearest millimeter.

A grandfather clock has a pendulum with a period of 2.00 s. Each "tic" and each "toc" is one second long. Based on your experiment what length of simple pendulum will have a period of 2.00 s? Is your answer reasonable based on the size of grandfather clocks?

What would be the period of a simple pendulum which has a length of 3.00 m?

More Graphs of Linear Equations

Linear equations are frequently written in the slope-intercept form: $y = mx + b$. This expresses y (the dependent variable) as a function of x (the independent variable). In this equation m is the slope of the line ($m = \Delta y/\Delta x$) and b is the y-intercept (the value of y when $x = 0$).

Given a linear graph and two points on the line, it is possible to determine the slope of the line and to use the point-slope form to write an equation for the line. This equation can then be rewritten in slope-intercept form.

Please do all exercises on a separate sheet of paper. Graphs should be done on graph paper.

Next we will look at some special cases of linear equations.

Special Case #1: If the y-intercept is zero ($b = 0$), then y is said to be directly proportional to x. The constant of proportionality is m, the slope of the corresponding graph of the line. Whenever the dependent variable is directly proportional to the independent variable the resulting graph will be a straight line passing through the origin.

Exercise #1: The mass of a piece of aluminum is directly proportional to its volume. The mass and volume of several pieces of aluminum are recorded in the table below.

a. Plot a graph of mass versus volume. Should you include the origin as a point on the graph? (Hint: What is the mass of a piece of aluminum with a volume of zero?)

b. Determine the slope of the graph. Include units. The slope is the proportionality constant which in this case is the density of aluminum.

c. Write an equation for the mass as a function of the volume.

Volume (cubic cm)	Mass (g)
1.00	2.75
1.50	4.00
2.00	5.40
2.50	6.75
3.00	8.15

Special Case #2: If the slope is zero ($m = 0$), then y is a constant ($y = b$). The value of y is independent of the value of x. This type of graph will not be of interest in your physics courses.

Special Case #3: If the slope of the graph is positive ($m>0$), y is a linearly increasing function of x. This means that when x increases, y also increases.

Exercise #3: Graph the equation $y = 3x + 5$.

Special Case #4: If the slope of the graph is negative ($m<0$), y is a linearly decreasing function of x. This means that when x increases, y decreases.

Exercise #4: Graph the equation $y = -3x + 5$.

Exercise #5: The height of the bottom of a hanging spring above a tabletop is measured as different weights are hung from the spring. This data is given in the table below.

a. Plot a graph of height above the tabletop as a function of the weight hung from the spring.

b. Calculate the slope of the resulting graph.

c. Write an equation relating the height of the bottom of the spring above the tabletop to the weight hung from the spring.

d. Is the height a linearly increasing or a linearly decreasing function of the weight?

e. According to your equation, what is the value of the height when no weight is hung from the spring? Does this agree with your graph?

f. Is the height directly proportional to the weight?

weight (lbs)	height (in)
1.00	36.00
1.50	34.25
2.00	32.50
2.50	31.00
3.00	29.15
3.50	27.50
4.00	25.75
4.50	24.10
5.00	22.25

Systems of Equations

<u>Multiple Equations with Multiple Unknowns</u>: The general rule that you need to be aware of is that to solve for two unknowns, you need two independent equations containing those two unknowns (and no other unknowns). More generally, you need n independent equations to solve for n unknowns (where n is any number—any positive integer, to be precise). The techniques for solving for the unknowns are illustrated below.

<u>Independent Equations</u>: If you can obtain one (or more) of your equations by algebraically combining the other equations, the set is *not* independent. This is not always easy to see, but it is important to be aware of. It will certainly come up in your physics courses.

Example 1: Is this set of equations independent or not? That is, can you get one of these equations as a combination of the other two equations?

(1) $A + B = C$ (2) $A - B = D$ (3) $2A = C + D$

This set is NOT independent. If you add equation (1) and equation (2), the result is equation (3). Thus, equation (3) provides no information beyond what is given by equations (1) and (2) and you cannot solve for three unknown variables using this set of equations. In fact, you should show that combining any two equations yields the third. Any two of these equations do make an independent set, so you could solve for two unknowns.

Exercise 1: For each set of given equations, how many independent equations are there? How many unknowns could you solve for?

(a) (1) $A + B = C$ (2) $A - B = D$ (3) $C + 3D = 7$

(b) (1) $V_1 - I_1 R_1 - I_2 R_2 = 0$ (2) $V_1 - I_1 R_1 - I_3 R_3 = 0$ (3) $I_3 R_3 - I_2 R_2 = 0$

<u>Solving Multiple Equations</u>: The general plan is to algebraically combine the equations so that you end up with one equation with only one unknown. You then solve for that unknown and use it to work back through your other equations to find the other unknowns. The two general methods for doing this are **substitution** and **elimination.**

General Procedure for Substitution Method

1. Solve one equation for one unknown in terms of the other(s).
2. Then substitute the result into another equation and solve it for the next unknown.
3. Continue until you have an expression for one unknown in terms of known quantities.
4. Then work backwards to find the other unknowns.

General Procedure for Elimination Method

1. Add one equation or a multiple of one equation to a second equation to eliminate one of the unknowns.
2. Repeat this process until you have one equation in one unknown and solve for that unknown.
3. It is also sometimes useful to divide one equation by another to eliminate one of the unknowns.

Let's look at a few examples that illustrate the methods that you will be using. We will first solve a system of equations by substitution to illustrate that method. For simplicity, we will first work with two independent equations and two unknowns.

Example 2: Solve the following system of equations for the unknowns B and C:

$$(1)\ A + B - C = 2D \qquad (2)\ B + 2C = D$$

Note that A and D are considered known quantities.

Step 1: Solve for one of the unknowns in terms of the other from either equation.
With practice, you learn to spot what might be the easiest approach.
In this case, we can solve for either B or C from either equation.
For this example, let's use equation (2) to get an expression for B.

Subtract $2C$ from each side of equation (2) and obtain: $B = D - 2C$

Step 2: Replace B in equation (1) with $D - 2C$. The result will be an equation with only C as an unknown.

$A + B - C = 2D$ becomes $A + (D - 2C) - C = 2D$ which is

$$A - 3C = D$$

Now isolate the unknown, C: $A - D = 3C$

And solve! $C = (A - D)/3$

Since A and D are known, now C is now known.

Step 3: Let's find B by substituting this expression for C back into ***either*** of the original equations ***or*** the equation we found at the end of Step 1.

At the end of Step 1, we found $B = D - 2C$. Substituting the expression in for C we obtain

$$B = D - 2[(A - D)/3] = D - \frac{2}{3}A + \frac{2}{3}D, \text{ so } B = \frac{5}{3}D - \frac{2}{3}A$$

(Notice that the -2 when distributed gave us a +2/3 D.)

We can now report the solution as $B = \frac{5}{3}D - \frac{2}{3}A$ and $C = \frac{A-D}{3}$.

The system of equations has now been solved. We were told that we already knew A and D, so we found expressions for B and C in terms of them. Any correct algebraic manipulations of the equations would have gotten us to the exact same results. But don't take my word for it, check it out yourself!

Exercise 2: For the same two equations, (1) $A + B - C = 2D$ and (2) $B + 2C = D$, solve for B and C using the following steps.

Step 1: From equation (1), find an expression for C.
Step 2: Substitute the expression for C found in step one in equation (2).
Step 3: Solve for B using the new form of equation (2).
Step 4: Substitute the expression for B found in step 3 into either original equation and solve for C.
Step 5: Compare your results to those obtained in the example. If they are not the same, one of us made a mistake.

Exercise 3: For the same two equations, solve for B and C by using the following steps.

Step 1: From equation (2), find an expression for C.
Step 2: Substitute the expression found in step one for C in equation (1).
Step 3: Solve for B using the new form of equation (1).
Step 4: Substitute the expression for B found in step 3 into either original equation and solve for C.
Step 5: Compare your results to those obtained previously.

Example 2 and Exercises 3 and 4 should illustrate for you that no matter which approach you choose, all the substitution methods yield the same solution.

Now that you've seen and practiced substitution, let's look at the elimination method. We will continue to work with the same system of equations and the same unknowns.

Example 3: Solve the following system of equations for the unknowns B and C:

$$(1)\ \ A + B - C = 2D \qquad (2)\ \ B + 2C = D$$

Step 1: To get rid of B, subtract equation (2) from equation (1). This is the same as multiplying equation (2) by negative 1and then adding the equations.

$$\begin{array}{cr} (1) & A + B - C = 2D \\ -1\times(2) & -B - 2C = -D \\ Sum & A - 3C = D \end{array}$$

Now we have an equation which does not contain the unknown B. We have **eliminated** B.

Step 2: Solve for C. It is easy to see that we obtain the same result as we did before.

$$C = (A - D)/3$$

Step 3: As before, substitute this expression into either equation and find B. The result, of course, will be identical to that obtained in all of the previous solutions.

Okay, your turn.

Exercise 4: Solve the same system of equations for B and C.

Step 1: Add a multiple of equation (1) to equation (2) to eliminate C.
Step 2: Solve the resulting equation for B.
Step 3: Substitute your expression for B back into equation (1) or (2) and solve for C.

The equations we encounter in physics are not always linear. The next example illustrates both methods applied when one equation contains a quadratic term with an unknown.

Example 4: Solve $x = x_0 + v_0 t + ½ a t^2$ and $v = v_0 + a t$ for v_0 and t.

Substitution: We could solve either equation for the unknown v_0 *or the unknown* t. *In general, we will want to choose to solve the simpler equation (in this case not the quadratic!) for one unknown. If we solve the second equation for* t, *we will have to substitute it into two places in the first equation* ***and*** *we would have to square it when substituting into the* $½ a t^2$ *term. If we solve instead for* v_0, *we will only have to substitute it into one place in the first equation (and not square it!). Here we go!*

1. Solve the second equation for v_0. $v_0 = v - a\,t$
   This can be done in 1 algebraic step! What algebraic step was required?
2. Substitute the result into the first equation. $x = x_0 + (v - a\,t)\,t + ½\,a\,t^2$
   We now have an equation with only one unknown!
3. Solve for t:

$$x = x_0 + v\,t - a\,t^2 + ½\,a\,t^2 = x_0 + v\,t - ½\,a\,t^2$$

$$x - x_0 - v\,t + ½\,a\,t^2 = 0$$

$$t = \frac{-(-v) \pm \sqrt{v^2 - 4\left(\tfrac{1}{2}a\right)(x - x_0)}}{2\left(\tfrac{1}{2}a\right)} = \frac{v \pm \sqrt{v^2 - 2a(x - x_0)}}{a}$$

4. Substitute the result for t back into one of the original equations or the equation from step 1.

$$v_0 = v - a\left(\frac{v \pm \sqrt{v^2 - 2a(x - x_0)}}{a}\right) = v - \left(v \pm \sqrt{v^2 - 2a(x - x_0)}\right) = \mp\sqrt{v^2 - 2a(x - x_0)}$$

The solution can now be reported as $t = \dfrac{v + \sqrt{v^2 - 2a(x - x_0)}}{a}$ and $v_0 = -\sqrt{v^2 - 2a(x - x_0)}$ or

$t = \dfrac{v - \sqrt{v^2 - 2a(x - x_0)}}{a}$ and $v_0 = +\sqrt{v^2 - 2a(x - x_0)}$.

Note, we have to correctly pair up our variables $t = \dfrac{v + \sqrt{v^2 - 2a(x - x_0)}}{a}$ and

$v_0 = +\sqrt{v^2 - 2a(x - x_0)}$ is **NOT** a solution.

Elimination:

1. Multiply the second equation by -t. $-v\,t = -v_0\,t - a\,t^2$
2. Add the result to the first equation.

$$\begin{aligned} x &= x_0 + v_0\,t + ½\,a\,t^2 \\ -v\,t &= -v_0\,t - a\,t^2 \end{aligned}$$

Sum: $x - v\,t = x_0 - ½\,a\,t^2$

Notice that the v_0 t and the $-v_0$ t cancelled.

Again, we have an equation with only one unknown!

3. Solve for t:

$$x - v\,t = x_0 - ½\,a\,t^2$$

$$x - x_0 - v\,t + ½\,a\,t^2 = 0$$

This is the exact same equation we applied the quadratic formula to solve in the previous method. From here the steps are the same as the substitution method. After you have t, substitute back into one of the original equations and solve for v_0

Now try your hand at these. Pick whichever method you like. If you're feeling ambitious, try a few different approaches for each. Note that those variables *not* identified as unknown are to be treated as known.

Exercise 5: Solve the system of equations (1) $\lambda + 2\mu = 11\omega$ and (2) $4\mu + \omega = 7\lambda$ for λ and μ. (Which variable is considered known?)

Exercise 6: Solve the system of equations (1) $y = 2x + 2$ and (2) $y = 8x - 1$ for x and y.

Once you have solved for x and y algebraically, carefully sketch the curves for each equation on the same graph. Find the point at which the curves intersect and compare with your algebraic result.

OK, the next exercise is here to illustrate a special case that sometimes comes up when solving systems of equations.

Exercise 7: Solve the system of equations (1) $2x - y = 0$ and (2) $y = 2x + 2$ for x and y.

First, see what happens when you solve algebraically by substitution or elimination. What does this mean? Can these two equations both be true at the same time?

After you have considered the problem algebraically, graph each equation and see if your algebraic findings make more sense to you.

So far the exercises you've been asked to do have turned out fairly simple. Real world problems are not always simple.

Exercise 8: Solve the system of equations (1) $2p^2t - 4r = 7$ and (2) $2t - 1 = r + 3$ for r and t.

Use substitution or elimination to solve.

Here is what you should find: $r = \dfrac{7 - 4p^2}{p^2 - 4}$ and $t = \dfrac{-9}{2p^2 - 8}$.

Now, this is not the only way the solutions can be written, if you get something different, check that you get these expressions if you write r and t as a single fraction.

More Systems of Equations #1

More systems of simultaneous equations

Most of the following are systems of equations you are likely to see in physics. Solve each system for the indicated unknowns. Assume any other variables in the equations are known quantities.

1.	(1) $x = v\,t$	(2) $x = \frac{1}{2}\,a\,t^2$	Solve for x and t.
2.	(1) $Mg - T = Ma$	(2) $T - mg = ma$	Solve for a and T.
3.	(1) $x = x_{10} + v_1 t$	(2) $x = x_{20} + v_2 t$	Solve for v_1 and v_2.
4.	(1) $F\cos\theta - \mu N = ma$	(2) $F\sin\theta + N - mg = 0$	Solve for N and a. Treat $F\cos\theta$ and $F\sin\theta$ as constants.
5.	(1) $x = v_{0x} t$	(2) $y = v_{0y} t + \frac{1}{2}\,g\,t^2$	Solve for t and y.

6. In Exercise 1(b) of the previous worksheet (Systems of Equations) you showed that any two of the equations

   (1) $V_1 - I_1R_1 - I_2R_2 = 0$ (2) $V_1 - I_1R_1 - I_3R_3 = 0$ (3) $I_3R_3 - I_2R_2 = 0$

   are independent. Assume all other quantities are known and solve for I_1 and I_2.

7. Any two of the above equations along with the equation

   (4) $I_1 = I_2 + I_3$ are independent. Use this to find I_1, I_2, and I_3.

8. In Exercise 1(a) of the previous worksheet (Systems of Equations) you showed that the equations

   (1) $A + B = C$ (2) $A - B = D$ (3) $C + 3D = 7$

   are independent. Solve this system for the unknowns A, C, and D.

9. Challenge Problem. Solve the following system of five independent equations for the five unknowns I_1, I_2, I_3, I_4, and I_5.

   (1) $I_1 = I_4 + I_5$ (2) $I_2 + I_3 = I_5$ (3) $V - I_1R - I_4(4R) = 0$

   (4) $2V + I_1(2R) - I_3(3R) = 0$ (5) $I_4(4R) - I_3(3R) = 0$

More Systems of Equations #2

Please do exercises on a separate sheet of paper and staple to this sheet.

Two Equations, Two Unknowns. Solve for the indicated variables.

1. a. Solve for x and t: (1) $x = v_s t$ (2) $x = \frac{1}{2} a_p t^2$.

   b. Find x and t when $v_s = 30.0$ m/s and $a_p = 2.44$ m/s^2.

2. Solve for a and T: (1) $m_1 g - T = m_1 a$ (2) $T - \mu m_2 g = m_2 a$.

3. Solve for *p* and *q*: (1) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ (2) $2p - 2q = 3f$.

Three Equations, Three Unknowns. Solve for the indicated variables.

1. Solve for F, T_1, and T_2: (1) $F - T_1 = m_1 a$ (2) $T_1 - T_2 = m_2 a$ (3) $T_2 = m_3 a$.

2. Solve for I_1, I_2, and I_3: (1) $I_3 = I_1 + I_2$ (2) $4I_1 - 4I_2 = 2$ (3) $4 = 4I_1 + 5I_3$.

3. Solve for a, α, and T_1: (1) $m_1 g - T_1 = m_1 a$ (2) $RT_1 = I\alpha$ (3) $a = R\alpha$.

More Systems of Equations #3

Solving Equations and Systems of Equations

Please do exercises on a separate sheet of paper and staple to this sheet.

A. Two Equations, Two Unknowns. Solve for the indicated variables.

1. a. Solve for t and h: (1) $v_y = v_{0y} - g\,t$ (2) $h = v_{0y}\,t - \frac{1}{2}\,g\,t^2$.

   b. Find t and h when $v_{0y} = 30.0$ m/s, $v_y = 0$ m/s ,and $g = 10.0$ m/s^2.

2. Solve for a and T: (1) $F - T - m_1 g = m_1 a$ (2) $T - m_2 g = m_2 a$.

B. Three or More Equations, Three or More Unknowns. Solve for the indicated variables.

1. a. Solve for μ in terms of T and w: (1) $T - f = 0$ (2) $n - w = 0$ (3) $f = \mu\,n$.
   (In other words, consider T and w to be the only known variables.)

   b. Find μ when T = 200 N and w = 500 N.

2. Solve for I_1, I_2, and I_3: (1) $V_1 - I_1 R_1 - I_3 R_3 = 0$ (2) $V_1 - I_1 R_1 + I_2 R_2 - V_2 = 0$
   (3) $I_3 = I_1 + I_2$

3. Solve for μ, T_1, and T_2: (1) $T_1 - \mu\,w_A = 0$ (2) $w_C - T_2 = 0$
   (3) $T_2 - \frac{3}{5}w_B - \frac{4}{5}\mu\,w_B - T_1 = 0$

   Evaluate when $w_A = w_B = 30.0$ N and $w_C = 39.6$ N.

4. Solve for I_1, I_2, I_3, I_4, and I_5: (1) $I_1 = I_4 + I_5$ (2) $I_5 = I_2 + I_3$
   (3) $V - I_1 R - I_4(4R) = 0$ (4) $2V + I_1(2R) - I_3(3R) = 0$
   (5) $I_4(4R) - I_3(3R) = 0$

Review of Systems of Equations and Graphs

Please do exercises and graphs on separate sheets of paper and staple to this page.

1. **Two Equations, Two Unknowns.**

Solve each system of equations for the indicated variables. Simplify your results.

a. Solve for x and y: (1) $3x + 2y = 6$ (2) $3x - 2y = 18$

b. Solve for T_1 and T_2: (1) $0.60\ T_2 - 0.80\ T_1 = 0$ (2) $0.60\ T_1 + 0.80\ T_2 = 100$

c. Solve for a and T: (1) $T - \mu mg = ma$ (2) $Mg - T = Ma$

d. Solve for t and x: (1) $x = v_0\ t$ (2) $y = \frac{1}{2} g\ t^2$

2. **Systems of Equations.**

Solve each system of equations for the indicated variables. Simplify your results.

a. Solve for a, N, and T:
(1) $F \cos\beta - \mu N - T = m_1 a$
(2) $N + F \sin\beta - m_1 g = 0$
(3) $T - m_2 g = m_2 a$

b. i. Solve for v_0 and V:
(1) $mv_0 = (m + M)V$
(2) $\frac{1}{2}(m + M)V^2 = (m + M)gh$

ii. Find values for v_0 and V if $h = 0.05$ m, $m = 0.005$ kg, $M = 1.0$ kg and $g = 9.8$ m/s^2

c. Solve for R_x, R_y, and T:
(1) $R_x - 0.80\ T = 0$
(2) $R_y - 0.60\ T - 800 = 0$
(3) $4.8\ T - 2000 = 0$

d. Solve for v_{1f} and v_{2f}:
(1) $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$
(2) $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

This is a challenging problem.

3. **Graphing Functions.**

Graph each of the given functions. These graphs can all be done on one sheet of graph paper. Use one-fourth of the page for each graph. Clearly label axes and scales used.

a. $y = 3x - 2$ b. $y = -3x + 2$ c. $2x - 3y = 6$ d. $x = 3t - 5$

4. **Finding Slopes of Lines.**

a. Find the slope of the line that passes through the points (4, 5) and (-2, 6).

b. Find the slope of the line that passes through the points (4, 5) and (4, 16).

5. **Writing Equations for Lines.**

a. Write an equation for the line that passes through the points (2, 3) and (3, 2).

b. Write an equation for the line with a slope of -3 and a y-intercept of 7.

c. Write an equation for the line with a slope of 5 that passes through the point (-2, 5).

6. Finding Slopes and Intercepts of Lines from Equations.

a. Find the slope and the y-intercept of the line whose equation is x + y = 6.

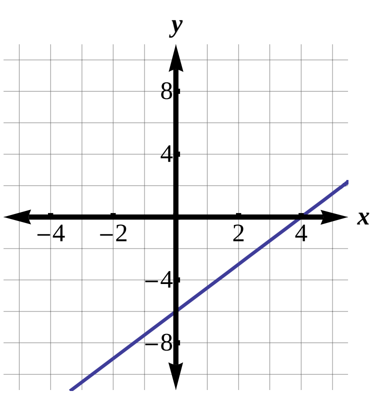
b. Find the x-intercept and the y-intercept of the line whose equation is 4x + 3y = 12.

c. Find the slope and the y-intercept of the line whose equation is 6x + 2y = 3.

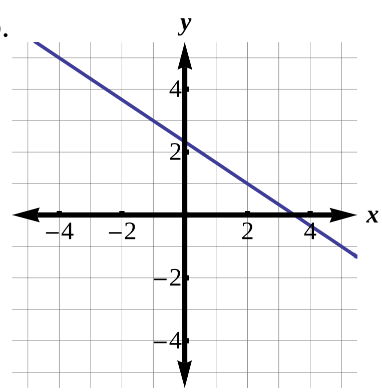
7. Finding Slopes and Equations from Graphs.

For each of the following graphs find the slope of the line and an equation for the line.

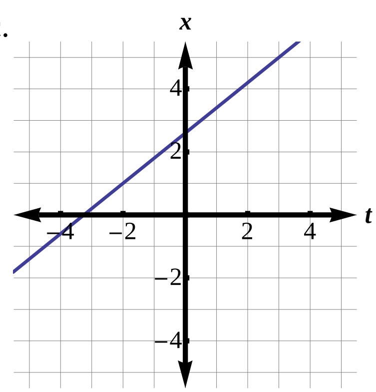
a.



b.



c.



8. Graphing Data and Interpreting the Results.

Given the following data, graph the data points and draw the curve that best fits the data (remember, a straight line is mathematically a curve). If the graph is a straight line, calculate the slope of the line and write an equation for the line. Both graphs may be drawn on the same set of axes. Use a different color for each graph and clearly indicate which graph goes with each set of data.

a. ___________________

time (s)	position (m)
0.00	0.0
1.00	1.0
2.00	3.9
3.00	9.1
4.00	15.9
5.00	25.2
6.00	36.1
7.00	48.9
8.00	64.0
9.00	82.0
10.00	99.5

b. ___________________

time (s)	position (m)
0.00	5.5
1.00	13.2
2.00	21.6
3.00	29.2
4.00	37.6
5.00	44.9
6.00	53.7
7.00	61.5
8.00	69.3
9.00	77.8
10.00	85.4

The two data sets are given for a block sliding along a smooth horizontal surface and a block sliding down a smooth inclined ramp. Which set of data represents which block?

Preparation for Problem Solving III
Problem Analysis

Problem analysis is different from problem solving. In problem analysis you do NOT try to solve the problem but rather to have a clear understanding of the problem. Hence, logically, problem analysis is a process that you need to go through before attempting to solve the problem.

Problem analysis consists of a set of steps (guidelines) you need to follow in order to have a clear "picture" of the problem. You should follow these steps for the problems given in this handout. The steps are general so you can apply them to future problems in this course or any future physics course you take.

Here are the steps:

(1) Read the problem from start to finish. You will refer back to the problem as necessary.

(2) Draw a picture or diagram of the problem, if possible.

(3) Ask yourself two questions:
  What quantities are given or implied? (What are the knowns?)
  What is it that the problem wants you to find? (What are the unknowns?)

(4) Assign a symbol (letter) to each quantity. Make sure different symbols are assigned to different quantities. If you have a diagram, label the quantities in your diagram with the symbols.

(5) Think about what principles, definitions and/or concepts are important in the problem and list them. Things like: "the area of a triangle" or "the definition of average velocity".

Now you should have a clear understanding of the problem.

Once more

DO NOT TRY TO SOLVE THE PROBLEM!!!

Example 1 will be done as in-class exercises. Each of the exercises should be done on a separate sheet of paper.

Example 1. A right triangle has a hypotenuse that is 1 cm more than twice one of its legs. The triangle has an area of 60 cm^2. What are the lengths of each side of the triangle?

Draw a diagram of the situation described in the problem.

What is the problem asking you to find?

What are the given (or implied) quantities?

Assign a symbol to each quantity above and label all the quantities in your diagram which can be labeled with these symbols.

List the names of the principles, definitions and/or concepts that are important in the problem.

Exercise 1. A 20-foot ladder leans against a vertical wall with the foot of the ladder 12 feet from the wall. A monkey hangs from the middle of the ladder. The monkey is 3.8 feet tall and his arms can reach 2.5 feet above his head. Can the monkey's feet touch the ground while he is hanging from the ladder?

Draw a diagram of the situation described in the problem.

What is the problem asking you to find?

What are the given (or implied) quantities?

Assign a symbol to each quantity above and label all the quantities in your diagram which can be labeled with these symbols.

List the names of the principles, definitions and/or concepts that are important in the problem.

Exercise 2. A rectangle has an area of 588 in^2 and a perimeter of 112 in. What are the length and width of the rectangle?

Draw a diagram of the situation described in the problem.

What is the problem asking you to find?

What are the given (or implied) quantities?

Assign a symbol to each quantity above and label all the quantities in your diagram which can be labeled with these symbols.

List the names of the principles, definitions and/or concepts that are important in the problem.

Exercise 3. On January 1, 2020 at 12:00 noon Eastern Standard Time, the people of the world will gather at the equator of the earth and form a line. They will tie a rope around the earth at the equator tightly and then loosen it by 20 feet. As a symbol of unity, they will try to crawl under the rope going from the northern hemisphere to the southern hemisphere, all at once. Assuming that the Earth is a perfect sphere with a radius of 4000 miles, will they be able to perform this ceremony?

Draw a diagram of the situation described in the problem.

What is the problem asking you to find?

What are the given (or implied) quantities?

Assign a symbol to each quantity above and label all the quantities in your diagram which can be labeled with these symbols.

List the names of the principles, definitions and/or concepts that are important in the problem.