Review of Scientific Notation and Significant Figures

Scientific Notation

Frequently numbers that occur in physics and other sciences are either very large or very small. For example, the speed of light in a vacuum is about 300,000,000 m/s while red light has a wavelength of approximately 0.000000650 m. Writing these numbers is simplified by using scientific notation. Scientific notation also allows numbers to be expressed in a form that clarifies the number of significant figures.

General form for scientific notation:

$$A \times 10^n,$$

where A is a number with a magnitude between 1 and 10 ($1 \leq |A| < 10$) and n is an integer.

To write the above numbers in scientific notation we can proceed as follows:

1) $300,000,000 \div 3.00 = 100,000,000 = 10^8$
So we can write $300,000,000 = 3.00 \times 100,000,000 = 3.00 \times 10^8$.
The speed of light is 3.00×10^8 m/s.

2) $0.000000650 \div 6.50 = 0.0000001 = 10^{-7}$
So we can write $0.000000650 = 6.50 \times 0.0000001 = 6.50 \times 10^{-7}$.
The wavelength of red light is about 6.50×10^{-7} m.

A shortcut for converting a number from decimal form to scientific notation is to move the decimal place a number of places that correspond to the appropriate power of ten. For example, using the numbers given above:

1) To convert 300,000,000 to a number between 1 and 10 we must move the decimal place 8 places to the left (corresponding to dividing by 10^8). To retain the same value, we must then multiply by 10^8.
So again, $300,000,000 = 3.00 \times 10^8$.

2) To convert 0.000000650 to a number between 1 and 10 we must move the decimal place 7 places to the right (corresponding to dividing by 10^{-7}). To retain the same value, we must then multiply by 10^{-7}.
So again, $0.000000650 = 6.50 \times 10^{-7}$.

We will also use scientific notation when multiplying or dividing very large or very small numbers. For example, the frequency of light is related to the speed of light and the wavelength of light by the formula
$$f = c/\lambda,$$
where f is the frequency, c is the speed of light, and λ is the wavelength. For red light

we can find the frequency as follows:

$$f = (3.00 \times 10^8 \text{ m/s})/(6.50 \times 10^{-7} \text{ m})$$
$$= (3.00/6.50)(10^8/10^{-7})(\text{m/s/m})$$
$$= (.462)(10^{15})\text{s}^{-1}$$
$$= 4.62 \times 10^{-1} \times 10^{15} \text{ s}^{-1}$$
$$= 4.62 \times 10^{14} \text{ Hz},$$

where 1Hz (one hertz) = 1 s^{-1}.

NOTE: The above example used the associative and commutative properties to rearrange and regroup terms. The rules of exponents were used to determine the correct power of ten. Frequently these steps are done mentally.

Decimal Places and Significant Figures

The result of the previous calculation was written with correct significant figures. Below are definitions and rules for using correct significant figures in calculations.

I. Definitions

The number of decimal places in a number is the number of digits to the right of the decimal point.

The number of significant figures in a number is the total number of digits, exclusive of leading zeros.

II. Examples

The following table shows the number of decimal places and the number of significant figures in five numbers.

Number	Number of Decimal Places	Number of Significant Figures
15.73	2	4
0.0072	4	2
200.6	1	4
1.2700	4	5
4300	0	2, 3 or 4

The ambiguity in the number of significant figures in the last example is easily removed by using scientific notation. 4.30×10^3 has three significant figures.

III. Rules for Rounding Off Calculated Results

In addition or subtraction, keep as many decimal places in the result as the smallest number of decimal places found in any of the numbers being added or subtracted.

Examples: $20.5 + 1.483 = 22.0$

                      $19.03 - 18.96 = 0.07$

                      $10.512 - 9.8 = 0.7$

                      $4.93 + 6.26 = 11.19$

Notice that the number of significant figures in the result can be more than the number of significant figures in either number or less than the number of significant figures in either number.

In multiplication or division, keep as many significant figures as the smallest number of significant figures found in any of the numbers being multiplied or divided.

Examples: $38.75(49.186)/1.48 = 1.29 \times 10^3$ or 1290, but NOT 1287.81.

                      $(1.237)(43.9)^2 = (1.237)\,(43.9)\,(43.9) = 2380$ or 2.38×10^3

Rules for Addition and Subtraction Using Scientific Notation

It is also important to be able to add and subtract numbers written in scientific notation. In order to do this, it is necessary for both numbers to be written using the same power of ten. (This may mean writing one of the numbers in a form that is not correct scientific notation.)

For example, suppose you need to add the thickness of two sheets of paper. One of the sheets of paper has a thickness of 8.25×10^{-4} m and the other has a thickness of 1.203×10^{-3} m. To add these two values, we must first convert the smaller number to a number written using the same power of ten as the larger number. Then we can add the two numbers.

$$(8.25 \times 10^{-4} \text{ m}) + (1.203 \times 10^{-3} \text{ m}) = (8.25 \times 10^{-1} \times 10^{-3} \text{ m}) + (1.203 \times 10^{-3} \text{ m})$$
$$= (0.825 \times 10^{-3} \text{ m}) + (1.203 \times 10^{-3} \text{ m})$$
$$= (0.825 + 1.203) \times 10^{-3} \text{ m}$$
$$= 2.028 \times 10^{-3} \text{ m}.$$

Most calculators use scientific notation and will convert between decimal notation and scientific notation as well as properly carrying out all calculations. It is important that you also be able to do these calculations by hand when necessary.

EXERCISES **(to be done without the use of a calculator)**

1. Which of the following are not written using correct scientific notation? In each case, explain why it is incorrect.

    a. 4.25×10^{-7} b. 63.3×10^{8} c. $1.64 \times 10^{-1.5}$ d. 8.02×10^{1}

2. Convert each number from decimal notation to scientific notation. Round all numbers to three significant figures.

    a. 3,020   b. 0.00000425   c. 0.0127

    d. 456,824,666   e. 2,450,000,000,000   f. 0.00000000036783

3. Convert from scientific notation to decimal notation keeping the same number of significant figures.

    a. 8.25×10^{-7} b. 8.25×10^{7} c. 6.01×10^{11}

    d. 4.68×10^{-13} e. 1.228×10^{-3} f. 8.987×10^{9}

4. Determine the number of significant figures in the following numbers.

    a. 86.5   b. 321.85   c. 20.0

    d. 0.931   e. 0.005   f. 0.0060

    g. 200   h. 2.00×10^{2} i. The integer 2

    j. π k. $\frac{4}{3}$ in $\frac{4}{3}\pi r^{3}$

5. Evaluate the following without the use of a calculator. Give your answers to the correct number of significant figures.

    a. $12.39 + 2.1$ b. 12.39×2.1 c. $0.4329 - 0.021$

    d. $0.4329 \div 0.021$ e. $43.2 + 61.4$ f. $993.2 + 8.295$

    g. $2.496 - 11.86$

6. Perform the following operations. Write all answers with correct significant figures. Be sure to write all answers using correct scientific notation.

a. $(8.00 \times 10^{74})(4.11 \times 10^{13})$

b. $(1.25 \times 10^{-151})(6.00 \times 10^{96})$

c. $(3 \times 10^{-215})(1.25 \times 10^{-215})$

d. $(7.52 \times 10^{16}) \div (4 \times 10^{12})$

e. $\dfrac{5 \times 10^{-15}}{1.25 \times 10^{-15}}$

f. $\dfrac{6.0 \times 10^{-30}}{4.0 \times 10^{12}}$

g. $\dfrac{(1.2 \times 10^{-3})(6.0 \times 10^{6})}{(4.0 \times 10^{2})}$

h. $\dfrac{(3.60 \times 10^{12})(5.0 \times 10^{-8})}{(2.50 \times 10^{-3})(7.20 \times 10^{14})}$

i. $(6.74 \times 10^{41}) + (2.5 \times 10^{39})$

j. $(4.298 \times 10^{25}) + (6.83 \times 10^{27})$

k. $(1.002 \times 10^{18}) - (9.45 \times 10^{17})$

l. $(5.73 \times 10^{-14}) - (3.8 \times 10^{-16})$

m. $(3.0 \times 10^{8})^{3}$

n. $(5.00 \times 10^{-3})^{2}$