

Review of Scientific Notation and Significant Figures

Scientific Notation

Frequently numbers that occur in physics and other sciences are either very large or very small. For example, the speed of light in a vacuum is about 300,000,000 m/s while red light has a wavelength of approximately 0.000000650 m. Writing these numbers is simplified by using scientific notation. Scientific notation also allows numbers to be expressed in a form that clarifies the number of significant figures.

General form for scientific notation:

$$A \times 10^n,$$

where A is a number with a magnitude between 1 and 10 ($1 \leq |A| < 10$) and n is an integer.

To write the above numbers in scientific notation we can proceed as follows:

- 1) $300,000,000 \div 3.00 = 100,000,000 = 10^8$
 So we can write $300,000,000 = 3.00 \times 100,000,000 = 3.00 \times 10^8$.
 The speed of light is 3.00×10^8 m/s.
- 2) $0.000000650 \div 6.50 = 0.0000001 = 10^{-7}$
 So we can write $0.000000650 = 6.50 \times 0.0000001 = 6.50 \times 10^{-7}$.
 The wavelength of red light is about 6.50×10^{-7} m.

A shortcut for converting a number from decimal form to scientific notation is to move the decimal place a number of places that correspond to the appropriate power of ten. For example, using the numbers given above:

- 1) To convert 300,000,000 to a number between 1 and 10 we must move the decimal place 8 places to the left (corresponding to dividing by 10^8). To retain the same value, we must then multiply by 10^8 .
 So again, $300,000,000 = 3.00 \times 10^8$.
- 2) To convert 0.000000650 to a number between 1 and 10 we must move the decimal place 7 places to the right (corresponding to dividing by 10^{-7}). To retain the same value, we must then multiply by 10^{-7} .
 So again, $0.000000650 = 6.50 \times 10^{-7}$.

We will also use scientific notation when multiplying or dividing very large or very small numbers. For example, the frequency of light is related to the speed of light and the wavelength of light by the formula

$$f = c/\lambda,$$

where f is the frequency, c is the speed of light, and λ is the wavelength. For red light

we can find the frequency as follows:

$$\begin{aligned}
 f &= (3.00 \times 10^8 \text{ m/s}) / (6.50 \times 10^{-7} \text{ m}) \\
 &= (3.00/6.50)(10^8/10^{-7})(\text{m/s/m}) \\
 &= (.462)(10^{15})\text{s}^{-1} \\
 &= 4.62 \times 10^{-1} \times 10^{15} \text{ s}^{-1} \\
 &= 4.62 \times 10^{14} \text{ Hz,}
 \end{aligned}$$

where 1Hz (one hertz) = 1 s⁻¹.

NOTE: The above example used the associative and commutative properties to rearrange and regroup terms. The rules of exponents were used to determine the correct power of ten. Frequently these steps are done mentally.

Decimal Places and Significant Figures

The result of the previous calculation was written with correct significant figures. Below are definitions and rules for using correct significant figures in calculations.

I. Definitions

The number of decimal places in a number is the number of digits to the right of the decimal point.

The number of significant figures in a number is the total number of digits, exclusive of leading zeros.

II. Examples

The following table shows the number of decimal places and the number of significant figures in five numbers.

Number	Number of Decimal Places	Number of Significant Figures
15.73	2	4
0.0072	4	2
200.6	1	4
1.2700	4	5
4300	0	2, 3 or 4

The ambiguity in the number of significant figures in the last example is easily removed by using scientific notation. 4.30×10^3 has three significant figures.

III. Rules for Rounding Off Calculated Results

In addition or subtraction, keep as many decimal places in the result as the smallest number of decimal places found in any of the numbers being added or subtracted.

Examples: $20.5 + 1.483 = 22.0$

$$19.03 - 18.96 = 0.07$$

$$10.512 - 9.8 = 0.7$$

$$4.93 + 6.26 = 11.19$$

Notice that the number of significant figures in the result can be more than the number of significant figures in either number or less than the number of significant figures in either number.

In multiplication or division, keep as many significant figures as the smallest number of significant figures found in any of the numbers being multiplied or divided.

Examples: $38.75(49.186)/1.48 = 1.29 \times 10^3$ or 1290, but NOT 1287.81.

$$(1.237)(43.9)^2 = (1.237)(43.9)(43.9) = 2380 \text{ or } 2.38 \times 10^3$$

Rules for Addition and Subtraction Using Scientific Notation

It is also important to be able to add and subtract numbers written in scientific notation. In order to do this, it is necessary for both numbers to be written using the same power of ten. (This may mean writing one of the numbers in a form that is not correct scientific notation.)

For example, suppose you need to add the thickness of two sheets of paper. One of the sheets of paper has a thickness of 8.25×10^{-4} m and the other has a thickness of 1.203×10^{-3} m. To add these two values, we must first convert the smaller number to a number written using the same power of ten as the larger number. Then we can add the two numbers.

$$\begin{aligned} (8.25 \times 10^{-4} \text{ m}) + (1.203 \times 10^{-3} \text{ m}) &= (8.25 \times 10^{-1} \times 10^{-3} \text{ m}) + (1.203 \times 10^{-3} \text{ m}) \\ &= (0.825 \times 10^{-3} \text{ m}) + (1.203 \times 10^{-3} \text{ m}) \\ &= (0.825 + 1.203) \times 10^{-3} \text{ m} \\ &= 2.028 \times 10^{-3} \text{ m}. \end{aligned}$$

Most calculators use scientific notation and will convert between decimal notation and scientific notation as well as properly carrying out all calculations. It is important that you also be able to do these calculations by hand when necessary.

EXERCISES (to be done without the use of a calculator)

1. Which of the following are not written using correct scientific notation? In each case, explain why it is incorrect.

- a. 4.25×10^{-7} b. 63.3×10^8 c. $1.64 \times 10^{-1.5}$ d. 8.02×10^1

2. Convert each number from decimal notation to scientific notation. Round all numbers to three significant figures.

- a. 3,020 b. 0.00000425 c. 0.0127
d. 456,824,666 e. 2,450,000,000,000 f. 0.00000000036783

3. Convert from scientific notation to decimal notation keeping the same number of significant figures.

- a. 8.25×10^{-7} b. 8.25×10^7 c. 6.01×10^{11}
d. 4.68×10^{-13} e. 1.228×10^{-3} f. 8.987×10^9

4. Determine the number of significant figures in the following numbers.

- a. 86.5 b. 321.85 c. 20.0
d. 0.931 e. 0.005 f. 0.0060
g. 200 h. 2.00×10^2 i. The integer 2
j. π k. $\frac{4}{3}$ in $\frac{4}{3}\pi r^3$

5. Evaluate the following without the use of a calculator. Give your answers to the correct number of significant figures.

- a. $12.39 + 2.1$ b. 12.39×2.1 c. $0.4329 - 0.021$
d. $0.4329 \div 0.021$ e. $43.2 + 61.4$ f. $993.2 + 8.295$
g. $2.496 - 11.86$

6. Perform the following operations. Write all answers with correct significant figures. Be sure to write all answers using correct scientific notation.

a. $(8.00 \times 10^{74})(4.11 \times 10^{13})$

b. $(1.25 \times 10^{-151})(6.00 \times 10^{96})$

c. $(3 \times 10^{-215})(1.25 \times 10^{-215})$

d. $(7.52 \times 10^{16}) \div (4 \times 10^{12})$

e. $\frac{5 \times 10^{-15}}{1.25 \times 10^{-15}}$

f. $\frac{6.0 \times 10^{-30}}{4.0 \times 10^{12}}$

g. $\frac{(1.2 \times 10^{-3})(6.0 \times 10^6)}{(4.0 \times 10^2)}$

h. $\frac{(3.60 \times 10^{12})(5.0 \times 10^{-8})}{(2.50 \times 10^{-3})(7.20 \times 10^{14})}$

i. $(6.74 \times 10^{41}) + (2.5 \times 10^{39})$

j. $(4.298 \times 10^{25}) + (6.83 \times 10^{27})$

k. $(1.002 \times 10^{18}) - (9.45 \times 10^{17})$

l. $(5.73 \times 10^{-14}) - (3.8 \times 10^{-16})$

m. $(3.0 \times 10^8)^3$

n. $(5.00 \times 10^{-3})^2$

Name: _____

Building Physical Intuition – Measurement 1

Groups: form groups of three

Equipment: each group should have a one-meterstick, a two-meterstick, and a tape measure

Lengths

Objectives: Estimates of human body dimensions. Developing familiarity with metric units

1. We will start by estimating and then measuring three body dimensions to get a feel for the units. Estimate each quantity in the following table to the nearest 10 cm. After making your estimates, use the one or two meter stick to measure each quantity and compare to your estimates. It is okay if these initial estimates differ from the measured values.

Partner 1's name _____ Partner 2's name _____

	Estimates to the nearest 10 cm	Measurements in centimeters
Your height		
Partner 1's height		
Partner 2's height		
Your palm width		
Partner 1's		
Partner 2's		
Your hand length		
Partner 1's		
Partner 2's		

2. Now that you have developed a feel for measurements in centimeters, complete the estimates of the quantities in the next table. After making your estimates, use the one or two meter stick to measure each quantity and compare to your estimates. Hopefully, your estimates show an improvement over the first set of estimates.

	Estimates to the nearest 10 cm	Measurements in centimeters
Your shoulder width		
Partner 1's		
Partner 2's		
Your nose to out-stretched finger tip		
Partner 1's		
Partner 2's		
Your leg length		
Partner 1's		
Partner 2's		
Your head height		
Partner 1's		
Partner 2's		
Your head width		
Partner 1's		
Partner 2's		

3. *Estimates of classroom dimensions.* Estimate each quantity to the nearest 1 m. Then use a tape measure to measure the length and the width of the classroom and two two-metersticks to measure the height of the classroom.

	Estimates to the nearest 1 m	Measurements in meters
Classroom width		
Classroom length		
Classroom height		

Units and Unit Conversions

Metric prefixes and some basic conversion factors are given in the Introduction.

The basic conversion between the metric system and the British system for lengths is the definition of the inch: 1 inch \equiv 2.54 cm. The conversion factor $\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)$ is equal to one so that multiplication by this conversion factor will change the units but not the value of the length being converted. (Note: the symbol \equiv means that this is a definition and therefore an exact relationship between the two values. Use of this conversion factor does not affect the number of significant figures in the length being converted. Conversions using the metric prefixes also do not affect the number of significant figures.)

4. *Units of Length*: There are ____ centimeters in a meter, ____ millimeters in a centimeter, ____ millimeters in a meter, ____ meters in a kilometer.

In order to convert from one set of units to another multiply by the appropriate conversion factors. Example: Calculate the number of millimeters in one foot:

$$1 \text{ ft} = 1 \text{ ft} \left(\frac{12 \text{ in}}{1 \text{ ft}}\right) \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) \left(\frac{10 \text{ mm}}{1 \text{ cm}}\right) = 304.8 \text{ mm}$$

You may use a calculator for the remaining problems on this worksheet.

5. Showing all conversion factors used, calculate the number of

a. miles in a kilometer

b. kilometers in a mile

c. yards in a meter

d. meters in a yard

Time

1. *Units of Time*: There are ____ seconds in a minute, ____ minutes in an hour, ____ hours in a day, ____ days in a year.

2. Showing all conversion factors used, calculate the number of

seconds in an hour:

seconds in a week:

minutes in a day:

seconds in a year:

Speed

The concept of speed (how fast an object is moving) will be discussed in more detail in later exercises. Speed is found by dividing a distance by a time interval to find the rate of motion. Therefore, common units for speed are mi/hr and m/s.

1. Convert a speed of 1 m/s to mi/hr. As usual, show all conversion factors used.

2. Convert a speed of 30 mi/hr to a speed given in km/hr and in m/s.

Name: _____

Lab Partners: _____

Building Physical Intuition – Measurement 2

Groups: form groups of three

Equipment: each group should have:

- a one-meterstick and meterstick caliper jaws
- a wooden block
- a wooden box

Measurements

1. Using the meterstick and meterstick caliper jaws measure the dimensions of the provided block. Record your measurements in the table below to the nearest 0.1 cm. You may choose which dimension to label length, width and height.

Length (cm)	
Width (cm)	
Height (cm)	

2. Using the meterstick and meterstick caliper jaws measure the **interior** dimensions of the provided wooden box. Record your measurements in the table below to the nearest 0.1 cm.

Length (cm)	
Width (cm)	
Height (cm)	

Calculations

For each calculation, show your work (what you are multiplying, dividing or adding) and round your answers to the correct number of significant figures.

1. Determine the volume of the provided block in cm^3 .

Name: _____

Lab Partners: _____

Building Physical Intuition – Measurement 3

Groups: form groups of three

Equipment: each group should have a metal block and vernier calipers. Groups will share triple-beam balances. Use the triple-beam balances at their given locations. Your instructor will demonstrate proper use of the vernier calipers and triple-beam balance.

Measurements

1. Using the **vernier calipers** measure the dimensions of the metal block. Do not assume that the length, width, and height are all the same. Record your measurements in the table below to the nearest 0.01 cm. You may choose which dimension to label length, width and height.
2. Use the triple beam balance to measure the mass of the block to the nearest 0.01 g. Your instructor will discuss proper use of the triple beam balance.

Length (cm)	
Width (cm)	
Height (cm)	
Mass (g)	

Calculations

For each calculation, show your work (what you are multiplying, dividing or adding) and round your answers to the correct number of significant figures.

1. Determine the volume of the provided block in cm^3 .
2. Determine the density of the provided block in g/cm^3 .
3. Convert the density to kg/m^3 .

Name: _____
 Lab Partners: _____

Building Physical Intuition – Kinematics

Kinematics is the *description* of motion. Motion can be described using the quantities: time, distance and displacement, speed and velocity, and acceleration. These exercises are designed to improve your observation and physical intuition for describing motion. Strive to improve your ability to make estimates of reasonable kinematics magnitudes.

Groups: form groups of three (or whatever number your instructor suggests)

Equipment: each group should have a one meter stick, a two meter stick, and a watch with a second hand and/or a stopwatch.

Distance vs. Displacement

There are important distinctions between what we mean by **distance** and **displacement**. In both cases, however, the amounts are measured in the same units as lengths.

	Amount	Direction
Distance	The amount traveled along a path.	Direction does <i>not</i> matter! It is not included in our definition.
Displacement	The length of the straight line from the earlier position to the later position, regardless of path.	Points from the earlier position straight to the later position.

1. *Measured Walks.* Place a small object on the ground to mark a starting spot (origin). (A piece of tape works well since it will stay in place.) Stand back to back with your partners so that you all walk at the same time, but in different directions. Without looking at your partners, walk straight ahead a distance that you think is 4 m. Mark your final position. Measure how far you walked from the origin. (Don't be concerned if you are off by quite a bit. This is a learning exercise to familiarize you with metric distances.) Enter your group's results in the table below.

	Person A	Person B	Person C
Experimental Attempt			
Ideal Distance			
Percent Error			

The **Percent Error** gives you a quantitative way to say how far off you were. Perform this calculation for each person as follows:

$$\text{Percent Error} = \frac{\text{Experimental} - \text{Ideal}}{\text{Ideal}} \times 100\%$$

Notice that the **Percent Error** is positive (+) if you walked too far (experimental exceeds the ideal) and is negative (–) if you did not walk far enough. Be sure to follow rules for significant figures

If everyone in your group walked exactly 4 meters, would your displacements all be the same? Be careful to check and use the definition for **displacement** when answering this question! _____, because _____

_____.

2. *Forward & Back Walks [1-D Motion]*. Have each member of the group take the walks described below. Place a small object on the ground to mark a starting spot (origin). Choose one direction to be the positive x-direction (+ x-direction). Put something on the ground to clearly show your choice for the + x-direction. The opposite direction is called the negative x-direction (- x-direction).

For each walk, you will: Sketch the walk showing the origin and positive direction. Measure and record the distances traveled for each of the stages. Also record the displacements for each of the stages including a +/- sign to indicate direction. Calculate the total distance and total displacement by addition. Measure all quantities to the nearest 0.01 m.

- WALK A: Walk 5 steps in the + x-direction and mark the spot. Walk 3 steps in the -x-direction and mark the spot.
- WALK B: Walk 5 steps in the -x-direction and mark the spot. Walk 3 steps in the +x-direction and mark the spot.
- WALK C: Make up your own 3-stage walk.

Stage 1: _____

Stage 2: _____

Stage 3: _____

	Person A	Person B	Person C
Walk A Sketch:	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m
Walk B Sketch:	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m	distance 1 = _____ m distance 2 = _____ m total dist. = _____ m displace. 1 = _____ m displace. 2 = _____ m total displ. = _____ m

	Person A	Person B	Person C
Walk C Sketch:	distance 1 = _____ m	distance 1 = _____ m	distance 1 = _____ m
	distance 2 = _____ m	distance 2 = _____ m	distance 2 = _____ m
	distance 3 = _____ m	distance 3 = _____ m	distance 3 = _____ m
	total dist. = _____ m	total dist. = _____ m	total dist. = _____ m
	displace. 1 = _____ m	displace. 1 = _____ m	displace. 1 = _____ m
	displace. 2 = _____ m	displace. 2 = _____ m	displace. 2 = _____ m
	displace. 3 = _____ m	displace. 3 = _____ m	displace. 3 = _____ m
	total displ. = _____ m	total displ. = _____ m	total displ. = _____ m

- Is distance ever negative? _____, because _____.

In the case of 1-D motion (motion along a line), the magnitude of a displacement is the absolute value of the displacement.

- In general, is the total distance equal to the magnitude of the total displacement? _____, because _____.
- In what special case is the total distance equal to the magnitude of the total displacement? _____.

Using walks A, B and C, find the average length of a single step for each person. Show how you used your data to obtain this result.

	Person A	Person B	Person C
Average Step Length:			

Method of calculation of average step length:

Speed vs. Velocity

The difference between the **average speed** of an object and the **average velocity** is effectively the same as the difference between **distance** and **displacement**. This is evident from the definitions:

- average speed $\equiv \frac{\text{distance}}{\text{time to travel that distance}}$
- average velocity $\equiv \frac{\text{displacement}}{\text{time for that displacement}}$

4. *From the definitions:*

- Does average speed have direction? _____ Why or why not? _____

_____.
- Does average velocity have direction? _____ Why or why not? _____

_____.
- Can average speed be negative? _____ Why or why not? _____

_____.
- Can average velocity be negative? _____ Why or why not? _____

_____.
- In general, is average speed equal to the magnitude of average velocity? _____ Why or why not?

_____.
- In what special case is average speed equal to the magnitude of average velocity? _____

_____.

5. *Human Speeds.* Along a straight line, mark a starting point (origin) and an ending point 10 m away. Choose one person to be a walker/runner, one person to be a timer with stopwatch, and the third person to be the recorder. The timer should be at the 10 m mark. Have the walker/runner start well behind the origin to get her/him up to a steady speed. When the walker/runner passes the origin, the timer should start the stopwatch.

Take turns being walker/runner, timer and recorder for the following different motions. Units are placed in the column headings and should not be included with numerical values you record in the table.

	Time (s)	Distance (m)	Displacement (m)	Ave. Speed(m/s)	Ave. Velocity (m/s)
slow walk					
normal walk					
fast walk					
jog					
run					

- In this special case where you do not change direction, what do you notice? _____

- To get a sense of how fast your speeds were, we can use the rough conversion of 1 m/s is about 2 mi/h. Using this rough conversion and the table above, fill in the blanks below. How many significant figures should you keep?

slow walking speed \approx _____ m/s \approx _____ mi/h

normal walking speed \approx _____ m/s \approx _____ mi/h

fast walking speed \approx _____ m/s \approx _____ mi/h

jogging speed \approx _____ m/s \approx _____ mi/h

running speed \approx _____ m/s \approx _____ mi/h

Do these numbers seem to be about right? _____.

6. *Race Walking!* In this contest you will take turns walking as quickly as you can—do not run or jog. Select and mark an origin. Choose a positive direction and measure 5.00 m from the origin in that direction. Clearly mark the spot. Measure and mark 10.00 m from the origin in the negative direction. Start from the origin, timer says "go," walk as fast as you can to the +5.00 m mark, turn around and walk as fast as you can to the finish line at the – 10.00 m mark.

Add units to the column headings and record your values in the table.

Name	Time ()	Distance ()	Displacement ()	Ave. Speed ()	Ave. Velocity ()

- Aside from the time to finish, which number would you use to determine the fastest walker? _____
 - What does the sign of the average velocity tell you? _____

 - What would your average velocity be if you started and stopped at the same spot?

- In that case, would your average speed be positive, negative or zero? _____

Simple Kinematics Problems: Distance/Displacement and Average Speed/Velocity

Below are some kinematics problems that describe some situations similar to those that you did in lab. Here you will not act out the situation but will use the descriptions to do calculations like those done in lab.

A. Define north to be the positive direction.

I take 7 steps north, moving a distance 6.59 m. Then, I take 4 steps south, moving an additional distance of 3.93 m.

1. Sketch my walk.
2. Compute the total distance I travel.
3. Compute my total displacement.
4. What is the magnitude of my total displacement?
5. Is the magnitude of my total displacement less than, greater than or equal to the total distance I travel?
6. Compute my average step length to the correct number of significant figures.

B. Define east to be the positive direction.

I drive my car 400.0 km east at 80.0 km/h. Then, I drive 150.0 km west at 50.0 km/h.

1. Sketch my drive.
2. What is the total displacement of the car I am driving?
3. What is the average velocity of the car during that entire displacement?
4. What is the total distance I travel?
5. What is my average speed for the entire trip?
6. Convert the average speed to m/s.

C. Define east to be the positive direction.

I drive my car 400.0 km east at 80.0 km/h. Then, I drive 400.0 km west at 50.0 km/h.

1. Sketch my drive.
2. What is the total displacement of the car I am driving?
3. What is the average velocity of the car during that entire displacement?
4. What is the total distance I travel?
5. What is my average speed for the entire trip?
6. Why is my average speed not $\frac{1}{2} (80.0 \text{ km/h} + 50.0 \text{ km/h}) = 65.0 \text{ km/h}$?