Applications

Physics requires the application of math skills to solve problems. In these exercises, we will apply the math concepts reviewed in this section to some problems.

Geometric Applications

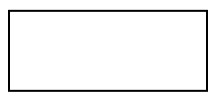
Some problems encountered in the real world require the knowledge of simple geometric formulas.

Example 1 Suppose you are asked to determine the width of a rectangle which has a length which is twice its width and has a perimeter of 18 feet.

To solve this we will follow a set of problem solving steps.

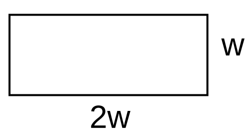
1. **Draw a picture**

   This problem is about a rectangle, so we draw a rectangle:



2. **Assign a symbol to each known and unknown quantity and label your diagram with those**

   Given: P = perimeter of rectangle = 18 ft
   Unknown: w = width of rectangle and $L = 2w$ = length of rectangle



3. **Determine the relationship between the known and unknown quantities**

   $P = 2L + 2w = 2*2w+2*w$ or $P = 2w + w + 2w + w$

4. **Solve symbolically**

   $P = 6w \rightarrow w = P/6$

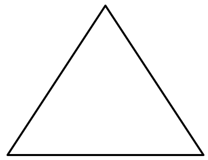
5. **Substitute known values and evaluate numerically**

   $w = P/6 = (18\text{ ft})/6 = 3\text{ ft}$

This very simple example illustrates our problem solving steps.

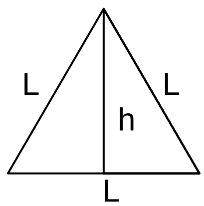
Example 2 Determine the area of an equilateral triangle with sides of length L.

1. Draw a picture



2. Assign a symbol to each known and unknown quantity and label your diagram with those

Given: L = length of each side; b = base of triangle = L
Unknown: h = height of triangle
Unknown: A = area of triangle



3. Determine the relationship between the known and unknown quantities

Considering the right triangle on the right with hypotenuse L and legs h and ½ L: $h^2 + (\frac{1}{2} L)^2 = L^2$.

$A = \frac{1}{2} b \cdot h = \frac{1}{2} L \cdot h$

4. Solve symbolically

Find h: $h^2 + \frac{1}{4} L^2 = L^2 \rightarrow h^2 = \frac{3}{4} L^2 \rightarrow h = \sqrt{\frac{3L^2}{4}} = \frac{\sqrt{3}\,L}{2}$

Compute A: $A = \frac{1}{2} L \cdot h = \frac{1}{2} L \cdot \frac{\sqrt{3}\,L}{2} = \frac{\sqrt{3}\,L^2}{4}$

5. Substitute known values and evaluate numerically

In this case there is no numerical value to substitute for L.

$A = \frac{\sqrt{3}\,L^2}{4}$ is the final result.

Exponential Growth and Decay Applications

In many areas of science as well as other fields, the growth rate of a quantity is proportional to its current value. For example, the growth rate of a population of buffalo is proportional to the number of buffalo present in the absence of any predators. In such a case, there is an exponential growth.

Example 3 Suppose the population of a herd of buffalo in Montana doubles every 3 years. Today, there are 20 buffalo in the herd. How many years will it take before there are 640 buffalo in the herd?

There is no picture to draw in solving this problem. So, we will begin our process by assigning symbols to the unknown and known quantities. Our problem solving steps are as follows:

1. Assign a symbol to each known and unknown quantity

Unknown: t = time for population to reach 640
Given: t_d = time for population to double = 3 yr
Given: N_0 = number of buffalo present today = 20
Given: N = number of buffalo present at time t = 640

2. Determine the relation between the known and unknown quantities

$$N = N_0 2^{t/t_d} = N_0 2^{\frac{t}{t_d}}$$

3. Solve symbolically

$$N = N_0 2^{t/t_d} \rightarrow 2^{t/t_d} = \frac{N}{N_0} \rightarrow \frac{t}{t_d} = \log_2\left(\frac{N}{N_0}\right) \rightarrow t = t_d \log_2\left(\frac{N}{N_0}\right)$$

4. Substitute known values and evaluate numerically

$$t = t_d \log_2\left(\frac{N}{N_0}\right) = (3\,\text{yr})\log_2\left(\frac{640}{20}\right) = (3\,\text{yr})\log_2(32) = (3\,\text{yr})(5) = 15\,\text{yr}$$

Many processes in science also involve exponential decay. Many chemical reactions have rates that are proportional to the amount of reactant present. For example, the decomposition of caffeine in the body occurs with a half-life of 6 hours. This means if you have 300 mg of caffeine present in your body at 8 am, you will have 150 mg present 6 hours later at 2 pm and 75 mg present after another 6 hours at 8 pm and so on.

<u>Example 4</u> Suppose you take a No-Doze pill which contains 200 mg of caffeine. How long will it take for the amount of caffeine in your body to drop to 25 mg?

1. **Assign a symbol to each known and unknown quantity**

   Unknown: t = time for amount of caffeine to reach 25 mg
   Given: $T_{1/2}$ = half-life of caffeine = 6 h
   Given: m_0 = beginning amount (mass) of caffeine = 200 mg
   Given: m = amount (mass) of caffeine present at time t = 25 mg

2. **Determine the relation between the known and unknown quantities**

$$m = m_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

3. **Solve symbolically**

$$m = m_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \rightarrow 2^{-\frac{t}{T_{1/2}}} = \frac{m}{m_0} \rightarrow -\frac{t}{T_{1/2}} = \log_2\left(\frac{m}{m_0}\right) \rightarrow t = -T_{1/2}\log_2\left(\frac{m}{m_0}\right)$$

4. **Substitute known values and evaluate numerically**

$$t = -T_{1/2}\log_2\left(\frac{m}{m_0}\right) = -(6\,\mathrm{h})\log_2\left(\frac{25\mu g}{200\mu g}\right) = -(6\,\mathrm{h})\log_2\left(\frac{1}{8}\right) = -(6\,\mathrm{h})(-3) = 18\,\mathrm{h}$$

<u>Exercises</u>

1. Determine the area of a square, which has a diagonal with length 3 m.

2. A rectangle has a length which is 3 feet less than twice its width. The area of the rectangle is 14 ft^2 (14 square feet). What is the length of the rectangle?

3. The population of rabbits in a county is found to triple every 4 years. If there are currently 100 rabbits in the county and this trend continues, how many rabbits will there be in 16 years?

4. Epinephrine (commonly called adrenaline) is a hormone released from the adrenal glands in emergency situations or when danger is sensed. Epinephrine has a half life of 2 minutes. If your body releases 400 micrograms of epinephrine when a deer jumps in front of the car you are riding in, how long will it take for the amount of epinephrine in your body to drop to 25 micrograms?