REVIEW OF ROOTS AND RADICALS

Definition: For any real number x and natural number n, the real number r is an <u>nth root</u> of x if $r^n = x$.

$\sqrt[n]{x}$	denotes the nth root of x.
x	is the radicand.
$\sqrt{\ }$	is the radical sign or the radical.
n	is the index, or order, of the radical.

Square roots: If x is a real number and x > 0,

$\sqrt{x}$	is the positive (prinicipal) square root of x.
$-\sqrt{x}$	is the negative square root of x.

If x is any real number, then $\sqrt{x^2} = |x|$.

Fractional exponents: The nth root of x can be denoted by $x^{1/n}$ or $\sqrt[n]{x}$.

Therefore, $x^{1/n} = \sqrt[n]{x}$.

For all positive integers n, $0^{1/n} = 0$.

If x is positive and n is an even positive integer, then $x^{1/n}$ is positive.
If x is positive and n is an odd positive integer, then $x^{1/n}$ is positive.
If x is negative and n is an even positive integer, then $x^{1/n}$ is not a real number.
If x is negative and n is an odd positive integer, then $x^{1/n}$ is negative.

RULES OF RADICALS

Product Rule: If x and y are any positive real numbers and n is a natural number, then

$$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy} \quad \text{or} \quad x^{1/n} \cdot y^{1/n} = (xy)^{1/n}.$$

Note: This rule is frequently used when working with quantities with units. Suppose you wish to know the length of an edge of a square with an area of 25 in^2. The length is related to the area by, $L^2 = A$, so $L = \sqrt{A} = \sqrt{25\,\text{in}^2} = \sqrt{25} \cdot \sqrt{\text{in}^2} = 5\,\text{in}$.

Quotient Rule: If x and y are any positive real numbers ($y \neq 0$) and n is a natural number, then

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}} \quad \text{or} \quad \left(\frac{x}{y}\right)^{1/n} = \frac{x^{1/n}}{y^{1/n}}.$$

Power Rule: If m and n are positive integers, then

$$x^{m/n} = \left(x^{1/n}\right)^m = \left(x^m\right)^{1/n}$$

provided all indicated roots exist.

If $x^{m/n}$ exists and x ≠ 0, then $x^{-m/n} = \dfrac{1}{x^{m/n}}$.

Since radicals can be written in fractional exponent form, these and other rules follow from the definitions and rules of exponents.

COMMONLY CONFUSED EXPRESSIONS

(1) $x^{1/n} \neq \dfrac{1}{x^n}$

(2) $\sqrt[n]{x+y} \neq \sqrt[n]{x} + \sqrt[n]{y}$

EXERCISES

1. Find examples that illustrate commonly confused expressions (1) and (2).

2. Without using a calculator, find the numerical equivalent of (that is, solve) each of the following.

a. $\sqrt{4} + \sqrt{25}$ b. $64^{1/2} - 36^{1/2}$ c. $\sqrt[3]{27} - \sqrt[4]{16}$

d. $125^{1/3} + 625^{1/4}$ e. $\sqrt[3]{8}\sqrt{9}$ f. $125^{1/3} \times 25^{1/2}$

g. $\dfrac{\sqrt[3]{100\,\sqrt[3]{10^{12}}}}{\sqrt[3]{10^9}}$ h. $(10^{27})^{-1/3} \times (10^{10})^{2/5}$

3. Simplify each expression if possible. Assume that all indicated roots exist.

a. $\sqrt{a} + \sqrt{b}$ b. $\sqrt[3]{\sigma} - \sqrt[4]{\sigma}$

c. $2\sqrt[3]{\sigma} - \sqrt[3]{\sigma}$ d. $2\sqrt[3]{\sigma} \cdot \sqrt[3]{\sigma}$

e. $\left(\sqrt[3]{\left(27x^2y\right)^6}\right)\left(\sqrt[3]{\left(8x^3y^6\right)^2}\right)$ f. $\left(\sqrt[3]{\left(6x^2y\right)}\right)\left(\sqrt[3]{\left(36x^4y^2\right)}\right)$

g. $\sqrt[3]{(\alpha+\beta)^6}\sqrt{(\alpha+\beta)^6}$ h. $\dfrac{\left(\sqrt[3]{(\lambda+\mu)^6}\right)(\lambda+\mu)}{\sqrt[3]{(\lambda+\mu)^9}}$

i. $\dfrac{\sqrt[3]{(\lambda+\mu)^6}+(\lambda+\mu)}{\sqrt[3]{(\lambda+\mu)^9}}$

j. $\left[(z+T)^{36}\right]^{-1/3} \times \left[(z+T)^{27/6}\right]^2$

k. $\dfrac{\left(\sqrt[3]{(a+b)^5}\right)\left(\sqrt[3]{(a+b)^4}\right)}{\sqrt[3]{(a+b)^{12}}}$

l. $\dfrac{\left(\sqrt[5]{(ab)^{51}}\right)\left(\sqrt[5]{(ab)^{21}}\right)}{\sqrt[5]{(ab)^{12}}}$

m. $\dfrac{\left(\sqrt[3]{\left(3p^2q\right)^5}\right)\left(\sqrt[3]{(3p^5q^2)^4}\right)}{\sqrt[3]{(p^6q^2)^2}}$

n. $\dfrac{\left(\sqrt[4]{\left(6x^2y^3\right)^2}\right)\left(\sqrt[4]{(3x^3y^4)^5}\right)}{\sqrt[4]{(12x^5y^2)^3}}$