

REVIEW OF EXPONENTS

Definition: If a is a real number and n is a natural number,

$$a^n = \underbrace{a \cdot a \cdot a \cdots}_{n \text{ factors of } a}$$

where a is the base and n is the exponent.

Definition: For any natural number n and any nonzero real number a , the negative exponent is defined by

$$a^{-n} = \frac{1}{a^n}.$$

RULES OF EXPONENTS

Product Rule: If a is any real number and m and n are natural numbers, then

$$a^m \cdot a^n = a^{m+n}.$$

Quotient Rule: If a is any nonzero real number and m and n are nonzero integers, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

Zero Exponent: If a is any nonzero real number, then $a^0 = 1$.

Note: 0^0 is undefined.

Power Rules: If a and b are real numbers and m and n are integers, then

$$\begin{aligned} (a^m)^n &= a^{m \cdot n} \\ (a \cdot b)^m &= a^m \cdot b^m \\ \left(\frac{a}{b}\right)^m &= \frac{a^m}{b^m} \quad (b \neq 0). \end{aligned}$$

Note: The power rule, $(a \cdot b)^m = a^m \cdot b^m$, applies when raising a number with units to a power. The volume of a cubical box that is 2 ft on each side is $(2 \text{ ft})^3 = 2^3 \text{ ft}^3 = 8 \text{ ft}^3$.

COMMONLY CONFUSED EXPRESSIONS

- (1) $-a^n$ and $(-a)^n$ do not mean the same thing.
- (2) ab^n is not the same as $(ab)^n$.
- (3) $(a + b)^n$ is not the same as $a^n + b^n$.

VERIFYING THE RULES OF EXPONENTS

The definitions of exponents and negative exponents may be used to verify the rules of exponents.

Example: The product rule.

$$\begin{aligned} a^m \cdot a^n &= \left(\underbrace{a \cdot a \cdot a \cdots}_m \right) \cdot \left(\underbrace{a \cdot a \cdot a \cdots}_n \right) \\ &= \underbrace{a \cdot a \cdot a \cdots}_{m+n \text{ factors of } a} \\ &= a^{m+n} \end{aligned}$$

Exercises A and B will be done in class. They should be included with the rest of the exercises from this worksheet.

EXERCISE A. Verify the quotient rule, the zero exponent rule, and the power rules using the definitions.

CHECKING THE COMMONLY CONFUSED EXPRESSIONS

If two expressions can be shown by any one example to yield different results, then the two expressions do not mean the same thing.

Example: (1) $-a^n$ and $(-a)^n$ do not mean the same thing.

We can illustrate this by choosing $a = 3$ and $n = 2$.

Now, $-a^n = -3^2 = -3 \cdot 3 = -9$, while $(-a)^n = (-3)^2 = (-3) \cdot (-3) = 9$

Since this example shows that the two expressions yield different results, the two expressions do not mean the same thing.

Note: You may be able to find examples where the two expressions yield the same result, even if the two expressions do not mean the same thing. You only need to find one example where the expressions yield different results to show that the expressions do not mean the same thing.

If $a = 3$ and $n = 3$,

then $-a^n = -3^3 = -3 \cdot 3 \cdot 3 = -27$ and $(-a)^n = (-3)^3 = (-3) \cdot (-3) \cdot (-3) = -27$

Although in this example both expressions yield the same result, the counterexample given above ($a = 3$ and $n = 2$) shows that the two expressions do not mean the same thing.

EXERCISE B. Find examples (numerical values for a , b and n) that illustrate commonly confused expressions (2) and (3).

EXERCISE C. USING THE RULES OF EXPONENTS.

Simplify each expression. Assume that all variables used as exponents represent integers and that all other variables represent nonzero real numbers. Write all answers with only positive exponents. Evaluate whenever possible.

1. a. $(-1)^1$ b. $(-1)^2$ c. $(-1)^3$ d. $(-1)^4$ e. $(-1)^5$ f. $(-1)^6$

g. Do you see a pattern in the signs here? Explain this result in words. Does this pattern only work for -1 or does it work for any negative number?

h. $(-1)^n = \underline{\hspace{2cm}}$ if n is odd. $(-1)^n = \underline{\hspace{2cm}}$ if n is even

j. $(-22)^n = 22^n$ if n is k. $(-22)^n = -22^n$ if n is

2. a. $x^3x^5x^2$ b. $(3x^4)(-4x^3)$ c. $(-5a^6b^4)(-2a^3b^5)$

3. a. 2^{-3} b. $(3p^3)^{-4}$ c. $-2z^{-2}$ d. $5(-2r^3)(3r^{-5})$

4. a. $\frac{4^7}{4^3}$ b. $\frac{y^3}{y^8}$ c. $\frac{t^{6n}}{t^{4n}}$ d. $\frac{(4b)^{12}}{(4b)^{12}}$

5. a. $\left(\left[(3x)^2 + 2x\right]^3 + x\right)^0$ b. $\frac{(3f^2)^4(2f^5)^2}{(9f^3)^6}$

6. a. $\frac{(-3\omega^2)^3(2\omega^{-3})^4}{(\omega^5)^{-6}}$ b. $\frac{\lambda^6 + \lambda^2 + \lambda^{-4}}{\lambda^3}$

7. a. $\frac{(p+q)^4(p+q)^2}{(p+q)^6}$ b. $\frac{3m^4(2m^5n^{-2})^2}{(4m^3)^3(6n^{-7})}$

8. a. $\frac{-(2c^3d^{-4})^5(-3^2cd^3)^3}{(-6^5c^{-4}d^6)^4}$ b. $\frac{(-8v^{-6}w^3)^{-2}(-4^2v^7w^{-2})^{-5}}{(2^5v^4w^3)^4}$