College of San Mateo
Official Course Outline

1. **COURSE ID:** MATH 280 **TITLE:** Proofwriting
   Units: 4.0 units **Hours/Semester:** 64.0-72.0 Lecture hours; and 128.0-144.0 Homework hours
   Method of Grading: Grade Option (Letter Grade or Pass/No Pass)
   Prerequisite: MATH 252
   Recommended Preparation:
   Eligibility for ENGL 110

2. **COURSE DESIGNATION:**
   Degree Credit
   Transfer credit: CSU

3. **COURSE DESCRIPTIONS:**
   Catalog Description:
   An introductory course on reading and writing proofs with an emphasis on mathematical rigor. Topics include formal logic, set theory, infinity, equivalence relations, well-orderings, modular arithmetic, the Euclidean algorithm; and proof techniques such as direct, indirect, contrapositive, induction, and exhaustion.

4. **STUDENT LEARNING OUTCOME(S) (SLO'S):**
   Upon successful completion of this course, a student will meet the following outcomes:
   1. Translate natural language propositions and arguments into symbolic form.
   2. Write a direct proof.
   3. Write an indirect proof.
   4. Write an inductive proof.
   5. Prove theorems in set theory.
   6. Prove theorems about algebraic structures.
   7. Use epsilon-delta arguments to prove statements about limits.

5. **SPECIFIC INSTRUCTIONAL OBJECTIVES:**
   Upon successful completion of this course, a student will be able to:
   1. Translate natural language propositions and arguments into symbolic form.
   2. Write a direct proof.
   3. Write an indirect proof.
   4. Write an inductive proof.
   5. Prove theorems in set theory.
   6. Prove theorems about algebraic structures.
   7. Use epsilon-delta arguments to prove statements about limits.

6. **COURSE CONTENT:**
   Lecture Content:
   1. Propositions and propositional forms
      A. Negation
      B. Conjunction
      C. Disjunction
      D. Truth tables
      E. Conditionals
      F. Biconditionals
   2. Soundness and validity
   3. Connectives and quantifiers
      A. Negating complicated statements.
      B. Epsilon-delta proofs for limits.
   4. Proof techniques
      A. Direct
      B. Indirect
      C. Induction and the well-ordering principle
      D. Other strategies (e.g. existence and uniqueness proofs, proof by exhaustion, “without loss of

generality," etc.)
5. Logical equivalences
   A. Double negation
   B. Converse and inverse
   C. Contrapositive
   D. Tautology
6. Axioms of Euclidean geometry and proofs of some geometric theorems.
7. Properties of the integers, the Well-Ordering Principle, the Euclidean Algorithm, modular arithmetic, Peano's axioms.
8. Sets, Relations, and Functions
   A. Union, intersection, subset, Cartesian product, power set, relative compliment
   B. Orderings (partial orderings, well-orderings), equivalence relations
   C. One-to-one correspondence, equipollence, the Schröder-Bernstein Theorem, finite and infinite sets, Cantor's diagonalization argument and an introduction to transfinite numbers.
   D. The Axiom of Choice, Zorn's Lemma, and the Well-Ordering Theorem
9. Algebraic structures such as groups, rings, fields, integral domains, homomorphisms, and isomorphisms.
10. An introduction to issues in foundations of mathematics such as constructive vs. non-constructive proofs, Gödel's theorem, the Löwenheim-Skolem "paradox," Category Theory, etc.

7. **REPRESENTATIVE METHODS OF INSTRUCTION:**
Typical methods of instruction may include:
   A. Lecture
   B. Critique
   C. Discussion
   D. Other (Specify): Moore Method or discovery-based learning (instructor supervises and engages with student presentations, no lecture, no outside books or resources, no outside help, no unsupervised collaboration, students develop independence and personal understanding of the material with minimal input from the outside.) Instructor may use a modified version of the Moore Method in which some of these rules are relaxed.

8. **REPRESENTATIVE ASSIGNMENTS**
Representative assignments in this course may include, but are not limited to the following:
Writing Assignments:

1. Students will submit written work containing their own original proofs and/or counterexamples.

2. Students may be asked to submit an organized portfolio of work.

9. **REPRESENTATIVE METHODS OF EVALUATION**
Representative methods of evaluation may include:
   A. Class Participation
   B. Class Work
   C. Exams/Tests
   D. Homework
   E. Oral Presentation
   F. Portfolios

10. **REPRESENTATIVE TEXT(S):**
Possible textbooks include:
   A. Day, M., V.. *Introduction to Proofs and the Mathematical Vernacular*, ed. Martin V. Day, 2016
   B. Chartrand, G.; Polimeni, A. D.; Zhang, P.. *Mathematical Proofs: A Transition to Advanced Mathematics*, 3 ed. Pearson, 2012

Other:
   A. Free open-source textbook link (Martin V. Day): http://www.math.vt.edu/people/day/ProofsBook

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Course Originator: Shawn Westmoreland